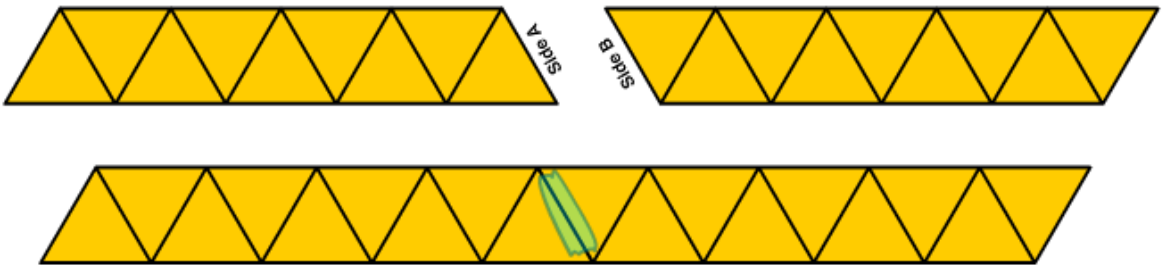
UW Math Circle

Week 5 – Origami

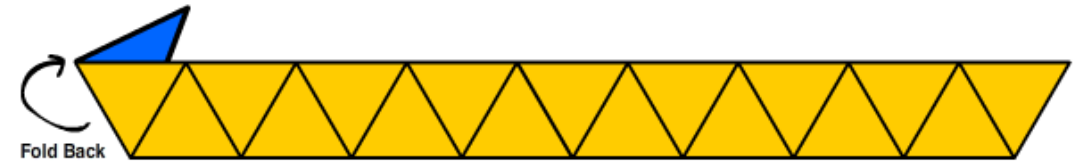
Hexaflexagon Folding Instructions

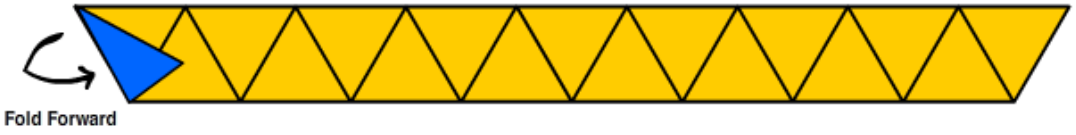
(In these diagrams, the yellow side is the front, and the blue side is the back.)

1. **Take the two pieces you've cut out, and tape "Side A" to "Side B".**

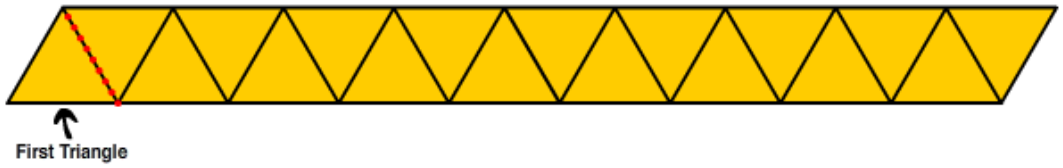


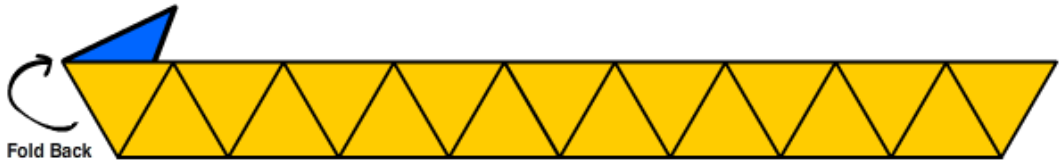
2. **Fold along all the black lines both ways (front and back) to make sharp creases.**



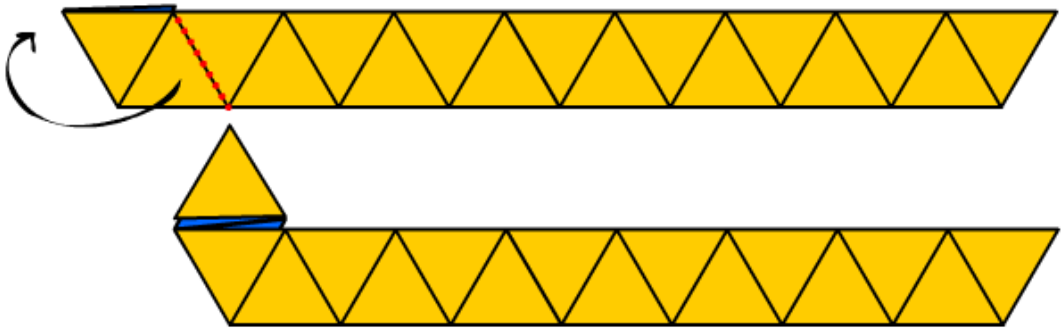


3. Now we need to wrap it around itself.
   Start by folding the FIRST triangle behind:

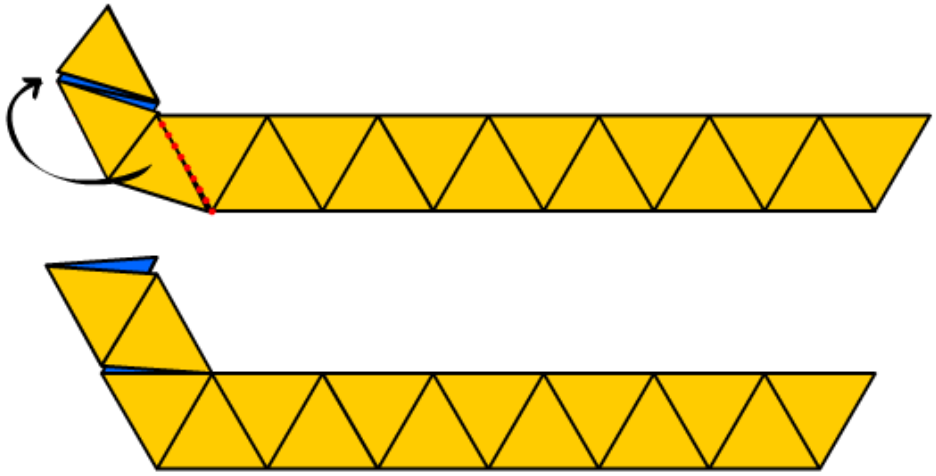




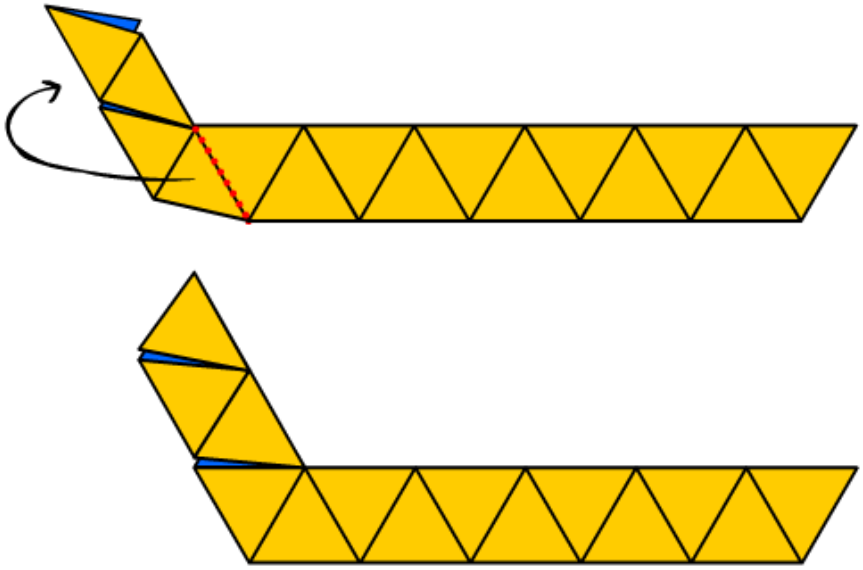
Then fold the first the next TWO triangles behind:



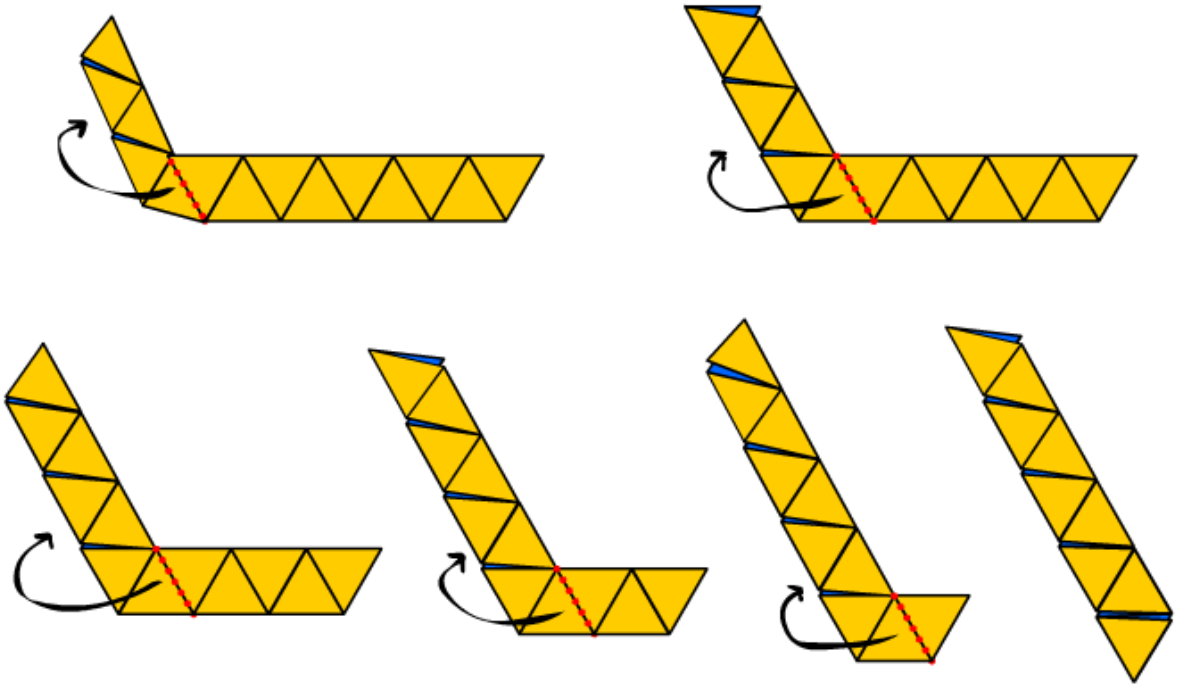
And the next two:



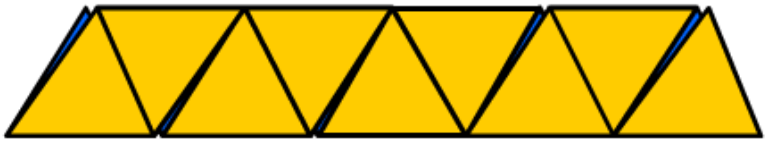
And the next two:



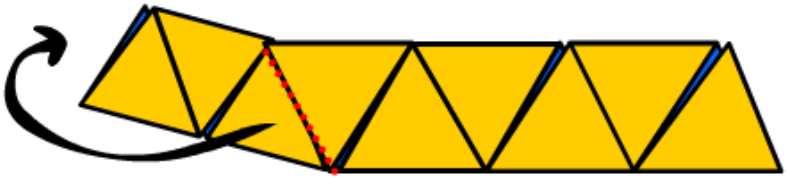
Continue wrapping it around itself in the manner.

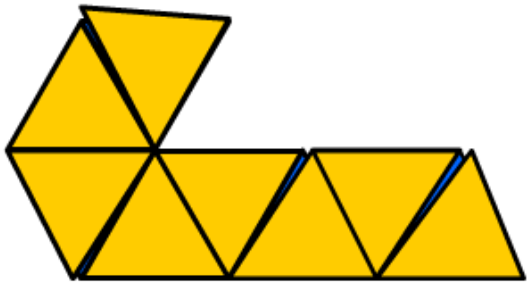


4. Once it is fully wrapped around itself, you should see NINE triangles:

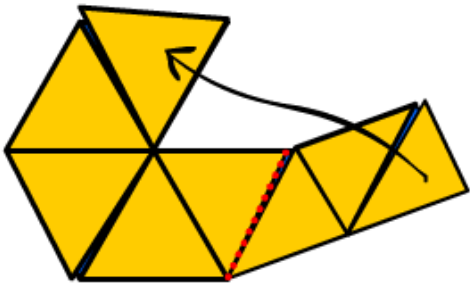


Take the first THREE, and fold them behind:

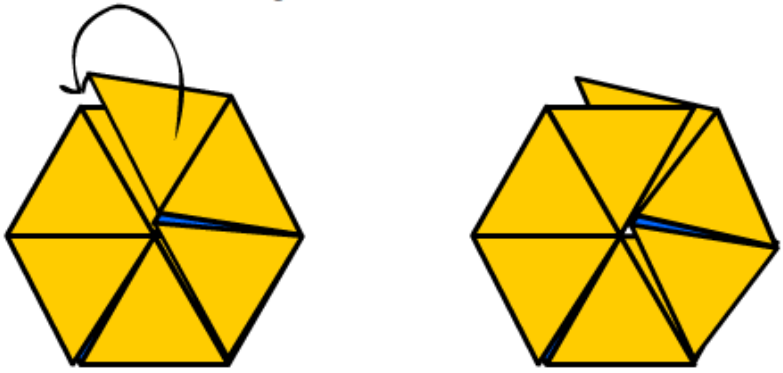




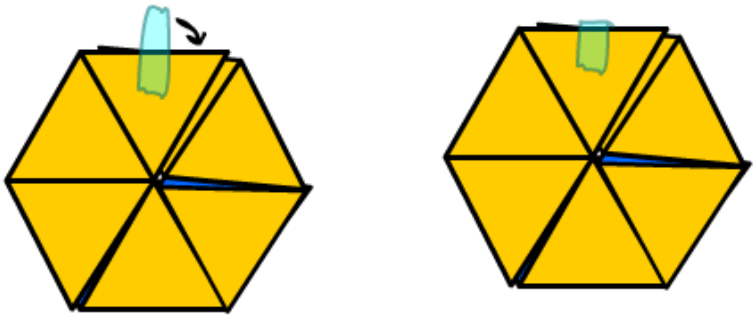
5. Take the LAST THREE, and fold them in front:



5. Now take the TOP triangle and tuck it BEHIND the other side.



6. On the TOP triangle, tape the FRONT to the BACK.



You're done! :-)

Instructions source: https://graceavery.com/hexaflexagon/

1 Hexaflexagons

1. Fold your hexaflexagon along with us. If you need help, let an instructor know!

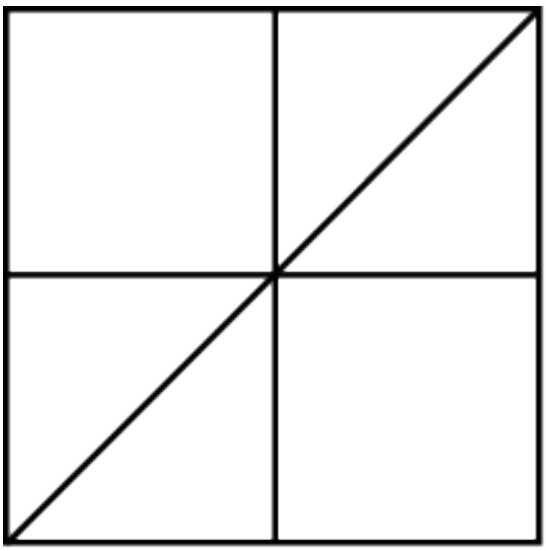
2. Your hexaflexagon has two visible sides: a front and a back. But when you fold it, new "sides" will be revealed! How many total sides does your hexaflexagon have? *Hint: decorate each side differently so you can tell them apart!*

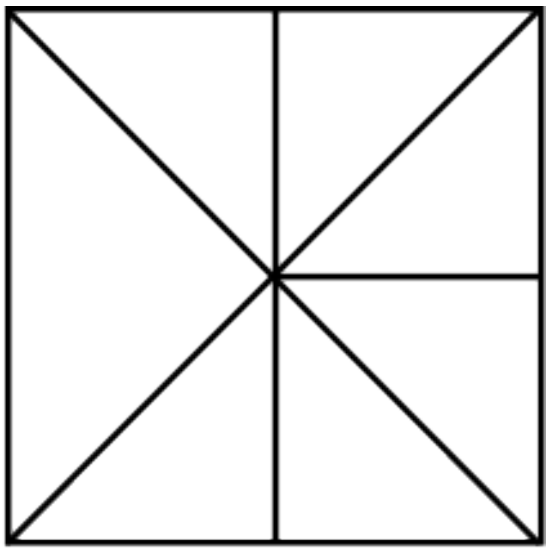
3. As you fold your hexaflexagon, you'll see its sides in different combinations. For example, let's say you colored three sides red, green, and blue. If your hexaflexagon is red in front and green on the back, then (red, green) is one possible combination. If you fold it, and now it's green in front and blue on the back, then (green, blue) is another possible combination.

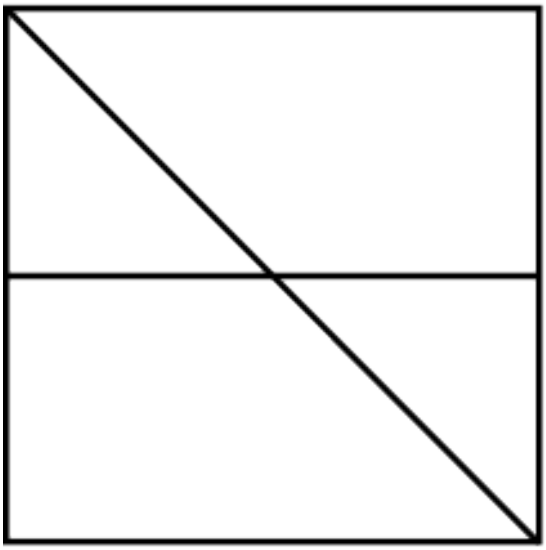
   How many different combinations can you find? *Note: some combinations might be impossible!*

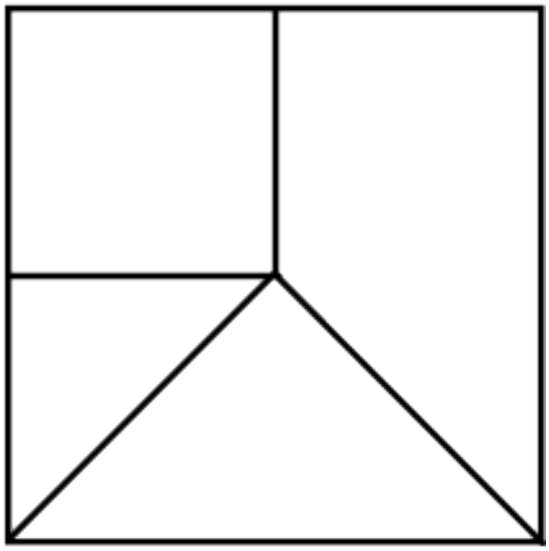
4. Are two appearances of a given face really the same? Draw a circle in the center of one face then try "flexing" your hexaflexagon. How many "flexes" does it take to return to the same orientation of a given face?

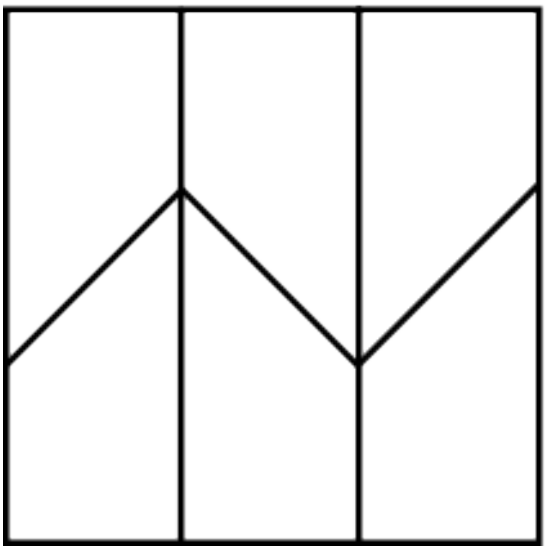
5. When the orientation of a face changes, how are the triangles making up the hexagon changing? Are they spinning in place? Are they moving with other triangles?

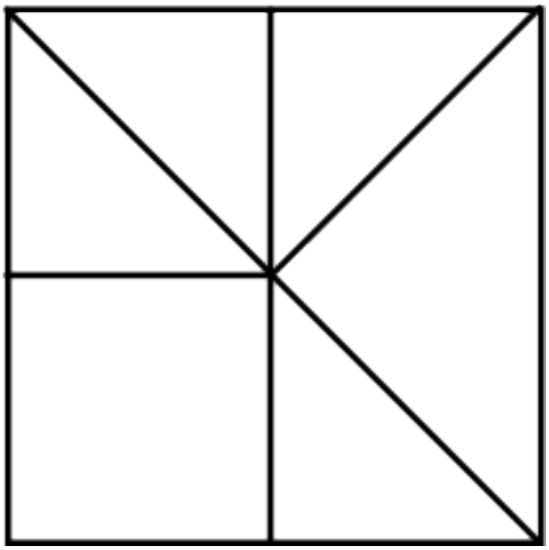






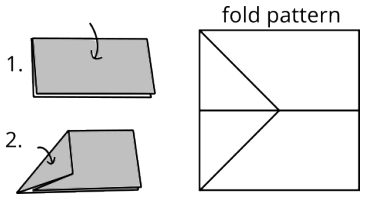






2 Fold Patterns

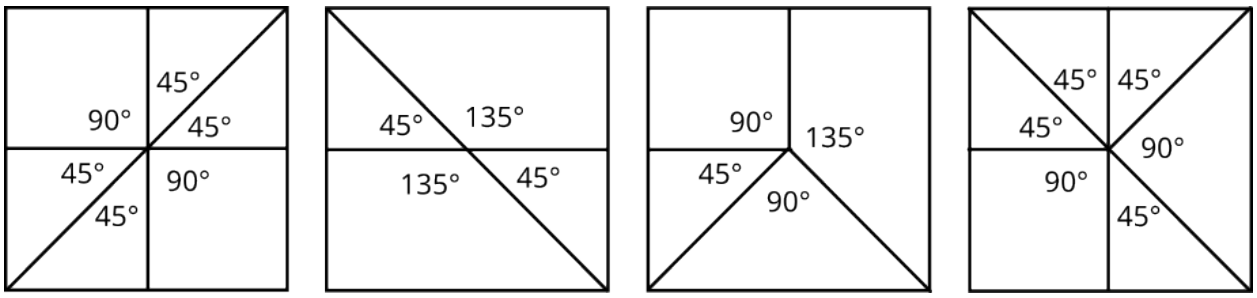
When you fold a piece of paper into any flat shape, the **fold pattern** is a picture of every fold line you used. If you fold along a line, but undo the fold later, that line doesn't count: the fold pattern is only the lines your shape is folded along *at the end*. **Example** $\longrightarrow$



6. Which fold patterns on page 7 are possible? In other words, can you fold a square paper into a flat shape using all the marked lines, but without using any other lines?

   For each fold pattern: if it's possible, how do you fold it? If not, why not?

7. Here are some of the fold patterns from Problem 6, with angles labeled. In patterns with all lines meeting at the center, can you find a general rule that determines whether the fold pattern is possible?



8. Notice that the first pattern in problem 7 is very similar to the second; if you rotate the second pattern 90° and add two horizontal line segments connecting the center to the edge, you get the first. Can you always take one of these radial patters (all line segments connect to one point) that is impossible and add line segments making it possible?

9. Consider the fold pattern on page 13. Is this fold pattern possible? Why or why not?

3 Construction Problems

10. You have a perfectly square piece of origami paper. Without any measuring implements, can you fold it into an equilateral triangle? It has to be exact—no estimating! For example, you can fold a point exactly onto another point or a line exactly onto another line, but you can't fold a 60° angle by eye alone.

11. How about a regular hexagon?