Name: ______________________________

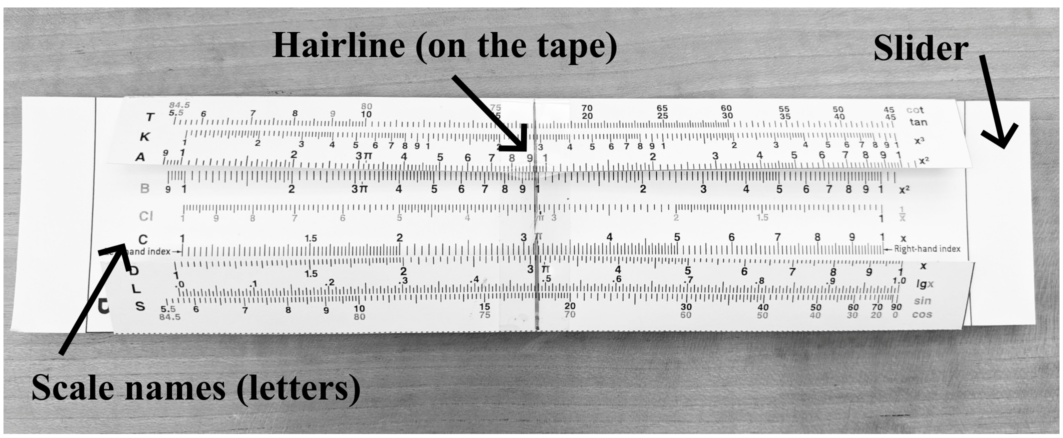
UW Math Circle

Week 15 – Slide Rules

Before the (digital) calculator was invented, every mathematician, engineer, and scientist carried a *slide rule* in their pocket. A slide rule is a kind of mechanical calculator that works like a fancy ruler. Slides rules were used well into the 1960s and 1970s — their calculations were even precise enough to be trusted by NASA engineers putting men on the moon! Today, we'll take a look at this piece of mathematical history and what it can teach us.

1 Getting to know your slide rule

Your Math Circle instructors have kindly made you a slide rule to use! Here are what the parts of a slide rule are called.



1. The scales all sort of look like rulers, but something is weird about them. What is unusual about them?

2. Find the C scale and CI scale on the slider. How are they similar and different?

3. Move the hairline to 1.2 on the C scale. Read the numbers on the CI scale and the B scale as accurately as you can.

   CI: ______________________________ B: ______________________________

2 Multiplication

Use your slide rule to follow along this tutorial.

To multiply 1.7×2.6,

- Align the left-hand 1 on the C scale with 1.7 on the D scale.
- Move the hairline to 2.6 on the C scale.
- Read the number on the D scale.

4. What number do you see? (Estimates are always fine, but be as accurate as possible.)

5. Use your slide rule to compute 3.8×2.1.

6. Use your slide rule to compute 1.13×1.26.

7. How could you use your slide rule to compute 17×330? Write down both your method and the answer.

8. How could you use your slide rule to compute 0.7×6? Write down both your method and the answer. (Hint: Look at the very right of your C scale. Why might that say 1, not 10?)

9. Use your slide rule to compute 289×812.

3 More features of your slide rule

10. One method to do division on your slide rule uses only the C and D scales that you've already been using. Figure out this method and write it down. (Hint: try to multiply $2 \times 3 = 6$. How can you compute that $6 \div 3 = 2$?) Then divide $113 \div 43$.

11. Another method to do division uses the CI scale and D scale. Using them to compute $a \div b$ is just like using the C and D scales to compute $a \times b$. That is,

    - Align one of the 1's on the CI scale with a on the D scale.
    - Move the hairline to b on the CI scale.
    - Read the number on the D scale, and adjust the decimal point if needed.

    Use this method to divide $589 \div 2.6$.

12. Use your favorite method to divide $0.184 \div 23$.

13. The B scale of your slide rule, used with the C scale, can help you find the square of a number. That is, for any number a, it can help you find $a \times a$. Play around with the slide rule to figure out how to do this, and be sure to mention how to interpret the numbers on the right half of the B scale.

14. Recall that the square root of a number a, denoted $\sqrt{a}$, is the number which, when squared, gives you a. For example, $\sqrt{9} = 3$. How can you use your slide rule to compute $\sqrt{41}$? Write down both your method and the answer.

15. Try to do multiplication (and/or division) using the A and B scales, just as if they were the same as the C and D scales. Does it always work? If so, what are the advantages and disadvantages to using the A/B scales over the C/D scales for multiplication? If not, when will it not work?

4 Doing longer calculations

Try to do all of the following problems **without writing down or memorizing any intermediate calculations**. Hint: the hairline will be super helpful!

16. $2.8 \times 1.6 \times 7.3$

17. $0.21 \div 54 \times 9.81$

18. $45 \times \sqrt{70}$

19. $3 \times \sqrt{\sqrt{5}}$

20. $1.9 \times \sqrt{3.8 \div (8.4 \times 2.3)}$

5 How does it work?

In order to really explain how a slide rule works, we need to first discuss an even more antiquated form of technology that came before it — the log table.

Recall that a^b means $\underbrace{a \times \cdots \times a}_{b \text{ times}}$. For example, $3^4 = 3 \times 3 \times 3 \times 3 = 81$.

21. Suppose a, b, and c are whole numbers. Why is it true that $a^b \times a^c = a^{b+c}$?

By using this fact, we can turn multiplication problems into addition problems! And although the basic definition of a^b only works when b is a whole number, it turns out that we can fill in the gaps to make the formula continue to work for decimals and fractions. People worked very hard to compute very accurate, giant tables to tell you for any given number a, if you wanted to write it as 10^b, what b should be. This is called a *table of logarithms*, and scientists used them for centuries before calculators were invented.

22. You know that $10^2 = 100$ and $10^3 = 1000$. Try to guess what number b makes $10^b = 500$.

23. In the table of logarithms provided for you in the back of this packet, find row 50, column 0, meaning 500. The number in the table is *the first 4 digits after the decimal point* of the number b. What is b? How close was your guess?

24. Using the log table, find the number b such that $10^b = 47$, and the number c such that $10^c = 673$.

b: ______________________________ c: ______________________________

25. Use the fact that $10^b \times 10^c = 10^{b+c}$, along with your answers to the previous question and your log table, to estimate the value of 47×673.

The first inventors of the slide rule relied on log tables to create their device.

26. Imagine an "addition slide rule" that just consists of two standard rulers. (Unfortunatly we don't have any to demonstrate, just try to imagine.) How can you use two rulers to add numbers, and why does that work?

27. Suppose you wanted to design a slide rule with just the C and D scales. Each scale should be 10 inches long, with the number 1 on both ends, as usual. You have a standard ruler and a log table. Where should you draw a mark for the number 2? Then, explain why this choice lets you use the C and D scales to multiply numbers.

28. What is the method used to label the CI scale? Use a fact similar to $a^b \times a^c = a^{b+c}$ to explain why this allows you to divide numbers, following the same process as multiplying on the C scale.

29. The A and B scales are constructed to be the square of the C and D scales. Based on this construction, explain why the left and right halves of the A and B scales ended up being exact copies of the C and D scales squished half as long. (And therefore, we can actually use the A and B scales to multiply and divide as well!)

Table of Logarithms

	0	1	2	3	4	5	6	7	8	9
10	0	43	86	128	170	212	253	294	334	374
11	414	453	492	531	569	607	645	682	719	755
12	792	828	864	899	934	969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996