Name: ______________________________

UW Math Circle

Week 13 – Logic

Today's circle is about mathematical logic. We will explore ways in which the language of logic helps us reason and solve some fun logic puzzles!

1 Truth Tables

1. If **A** is a statement, then ∼**A** is the exact opposite of statement **A**.
   *e.g. If **A**="I like ice cream.", then ∼ **A**= "I do not like ice cream."*

   Try it for yourself!

   If **A** = "Dragons are scary."

   ∼**A** = ____

   If **B** = "Dragon breath is not smelly."

   ∼**B** = ____

   If **C** = "Dragons always do their homework."

   ∼**C** = ____

2. If statement **A** is true, what do we know about ∼**A**?

3. We can record this information in a "Truth Table". Fill in the rest of the table based on whether **A** is true, "T", or false, "F".

A	∼**A**
T	
F	

4. If **A** and **B** are statements, then "**A** and **B**", denoted **A**∧**B**, is true only when BOTH **A** and **B** are true and "**A** or **B**", denoted **A**∨**B**, is true if EITHER **A** or **B** are true. Try it yourself!

   If **A** = "Bubblegum is delicious." and **B** = "Bubble gum bubbles should only be blown on the weekend."

   A∧**B** = ______________________________

   A∨**B** = ______________________________

   ∼(**A**∨**B**) = ______________________________

5. Now, try filling out the blanks of the following Truth Table.

A	**B**	∼**A**	∼**B**	**A**∧**B**	**A**∨**B**	∼(**A**∨**B**)	(∼**A**)∧(∼**B**)
T	T						
T	F						
F	T						
F	F						

   Have your an instructor check your work!

6. What do you notice about the final two columns?

7. Two logical statements are equivalent if their Truth Tables are identical.

   Edward tells Joanne "I do not like blueberries or strawberries." and he tells Taylor "I do not like blueberries and I do not like strawberries." Did Edward give the same information to Joanne and Taylor? Decide what **A** and **B** are to use the Truth Table in question 5 to check.

Try some puzzles! *(Hint: If you get stuck try making a truth table to try out different strategies!)*

8. Two mathematicians are trapped on an island. Every day, they each flip a coin, and guess the other's result. If at least one of the mathematicians is correct, the island blesses them with delicious fruit; if they are both wrong, the island will stop producing fruit leaving them to survive off kale indefinitely. Is there a strategy which spares the trapped islanders from leafy greens?

9. The two mathematicians learn that if they both guess each other's coin flip result correctly on the same day, then they will be freed from the island. Is there a strategy which can get them freed in a limited number of days without risking having to eat kale indefinitely?

Sometimes one logical statement can imply another!

For example, "If Bart plays Minecraft during class, then Bart will not understand the homework."

In this example if **A** = "Bart plays Minecraft during class" is true, then **B** = "Bart will not understand the homework." We denote this **A**→**B**.

Here is the truth table:

A	**B**	**A**→ **B**	∼(**A**→ **B**)
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

The last two rows might seem puzzling at first. However, if Bart does not play Minecraft in class then the implication that Bart will not understand the homework is not broken! Additionally, for considering the fourth row, the statement "If Bart did understand the homework, then Bart did not play Minecraft during class." is an equivalent statement to **A**→**B**.

10. Try this puzzle:

At least one of your three kittens (Skittles, Fluffy, and Jumino) has eaten your fresh caught salmon.

(a) If one of either Skittles or Jumino ate the salmon, then Fluffy ate it with them.

(b) Skittles and Jumino never eat together.

Is it possible that Skittles did NOT eat any salmon? Is it possible that Fluffy did NOT eat any salmon? What about Jumino?

2 Lying Puzzles

You have just arrived on an island inhabited by two kinds of people: knights who always tell the truth, and knaves who always lie. Every inhabitant of the island is either a knight or a knave. Knights and knaves dress the same way, so when you meet a stranger you can't immediately tell if they are a knight or a knave.

11. You come across two inhabitants of this island, A and B. A makes the following statement: "At least one of us is a knave." What can you determine about A and B?

12. This time, A instead states, "Either I am a knave or B is a knight." What can you determine about A and B?

13. You come across three inhabitants of this island, A, B, and C. A and B make the following statements:

    A: All of us are knaves.
    B: Exactly one of us is a knight.

    What can be determined about A, B, and C?

14. With three inhabitants again, A, B, and C. A states, “B and C are of the same type.” You then ask C, “Are A and B of the same type?” Is it possible to deduce what C will answer?

15. You come across three particularly strange inhabitants, A, B, and C. You do not speak their language, but you know that the words da and ja are their words for yes and no. You do not know which word means which. In addition, C is not a knight or a knave, but rather a normal. Normals sometimes lie and sometimes tell the truth. You can ask exactly three yes or no question; each to exactly one inhabitant.

    What can you ask to determine the identities of A, B, and C?