

UW Math Circle

Week 17 – Puzzles!

Hat Puzzles. Before starting the puzzles below, make some oragami hats with your group.

1. We will play the following game. Everyone in your group will form a line and your instructor will place either a blue hat or a yellow hat on each student's head. You can see the color of the hat of everyone standing in front of you, but not your own and no communication between you is allowed. One at a time starting from the back of the line, each student will guess the color of their own hat. If at most one student guessed the wrong color hat, you win. Can you find a strategy that guarantees a win?
2. Next, three students in your group will form a circle (if you are in a group of 4, take turns) and your instructor will place a colored hat on each of your heads. Each hat will be either blue, yellow, or green. You will be able to see the color of the hat of everyone else in your group, but not your own and no communication is allowed. On the count of 3, you will all guess the color of the hat you are wearing and if any one of you are right, you win. Can you find a strategy that guarantees a win?
3. Three students in your group will form a line (if you are in a group of 4, take turns) and your instructor will place either a blue or yellow hat on each of your heads. However, these hats will not all be the same color. At any time, each student may guess their color hat. If one student correctly guesses the color of their hat, you all win but if any student makes an incorrect guess, you all lose. Can you find a strategy that guarantees a win?
4. Form a circle with 4 or 6 students (if you are in a group of 3, you will need to combine with a neighboring group). Then an instructor will place either a blue or yellow hat on each student's head. On the count of 3, you will all guess the color of your own hat. If at least 50% of you are correct, you win. Can you find a strategy that guaranteed a win?

Scale logic puzzles

5. Suppose you have 9 coins, but one is counterfeit and will be slightly lighter than the others. You are given a scale (shown below) to compare the weights of the coins. Can you find a strategy that is guaranteed to find the fake coin by using the scale only twice?



6. You have six stones, two colored red, two colored yellow, and two colored green. Aside from their colors, all six stones look and feel exactly the same. Unfortunately, in each colored pair, one stone is slightly heavier than the other. All the lighter stones have the same weight, and all the heavier stones have the same weight. Using a balance scale to make two measurements, decide which stone of each color is lighter.
7. Pirate Henry had 8 boxes of gun powder weighing 1, 2, 3, 4, 5, 6, 7, and 8 pounds. The weight is printed on top of every box. Pirate Bob hid a 1-pound gold bar in one of these boxes. Pirate Henry has a balance scale that he can use, but he cannot open any of the boxes. Help him find the box with the gold bar using two weighings on the balance scale.

Truth vs Lying Logic Puzzles

8. Princess Peach meets a goomba and piranha plant in the forest. The goomba lies every Monday, Tuesday and Wednesday and the other days he speaks the truth. The piranha plant lies on Thursdays, Fridays and Saturdays, and the other days of the week he speaks the truth. “Yesterday I was lying,” the goomba told Princess Peach. “So was I,” said the piranha plant. What day is it?

9. Boo, Bob-omb and Blooper have each been working a second job. One is a knight, one a knave, and one a spy. The knight always tells the truth, the knave always lies, and the spy can either lie or tell the truth. Boo says: “Blooper is a knave.” Bob-omb says: “Boo is a knight.” Blooper says: “I am the spy.” Who is the knight, who the knave, and who the spy?

10. Daisy meets three koopas on a mountain top. One always tells the truth, one always lies, and one tells the truth or lies randomly. We can call them Truth, False and Random. They understand English but answer in their own language, with ja or da for yes and no—but you don’t know which is which. You can ask three questions to any of the koopas (and you can ask the same koopa more than one question), and they will answer with ja or da. What three questions do you ask to figure out who’s who?

11. An infinite line of people $1, 2, 3, \dots$ are standing wearing hats, each with a real number on their hat. They are asked to guess the number on their hat simultaneously. Give a way for all but a finite number of people to guess correctly.
(Assume people have infinite memory. Assume axiom of choice: given some collections, they can agree on a way to pick one item from each collection.)
Follow-up: Now they guess in order. Give a way for at most 1 person to guess incorrectly.