Week 6 — *Presidents' Valentine's Day Auction!*

AUCTION RULES

- Find the best answers that you can to these questions.
- The following are ALLOWED: calculators, the internet
- The following is NOT allowed: writing computer programs
- Each team should pick a team name and a team captain.

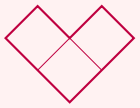
The auction will start at 6:20 PM!

Question 1. Find the largest prime number that can be written as the sum of two square numbers.

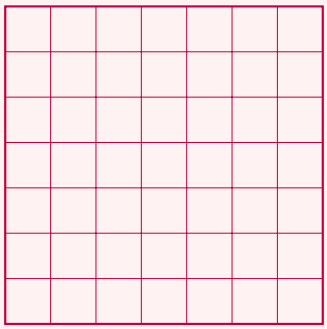
For example, 5 is a prime number, and it can be written as $5 = 1^2 + 2^2$.

(A prime number is a number with no factors except 1 *and itself. A square number is a number equal to an integer to the power of* 2*.)*

Question 2. Here is a heart and a chocolate:



Completely fill this grid with hearts and chocolates to give to your valentine. (You can rotate the hearts.) You may not put two chocolates next to each other, horizontally or vertically (they'll melt into each other and make a mess).



Your valentine is hungry — find an arrangement with as many chocolates as possible.

(Note: You might want to take a screenshot of your best solution, to make it easier to show in the auction.)

Question 3. Find a president's name that has the highest score in Scrabble.

The name must be a first name, last name or middle name of a president — no spaces, punctuation marks or other characters except letters A–Z. The president can be a current or former president of any country.

For example, "JOE" scores 10 points in Scrabble (J = 8, O = 1, E = 1).

Question 4. The President of Mathland has invited the numbers from 1 to 100 to a Valentine's Day ball, and they're performing the Euclidean Dance. In this dance, numbers form pairs, with the smaller number on the left and the bigger number on the right. In each step of the dance, the smaller number is subtracted from the bigger number. If this causes the bigger number to become smaller than the other number, the numbers do-si-do and switch places. The dance ends when the numbers become equal.

For example, the pair 6 and 14 dance like this:

$$6, 14 \;\to\; 6, 8 \;\to\; 6, 2 \;\rightsquigarrow\; 2, 6 \;\to\; 2, 4 \;\to\; 2, 2$$

What pair of numbers between 1 and 100 can you find that has to do-si-do as many times as possible?

Question 5. Find the longest list of numbers from 1 to 100 where every number in the list is a factor of the product of the previous numbers plus 1.

For example, $5, 3, 4$ is a valid list of length three: 3 is a factor of $5 + 1$ and 4 is a factor of $(5 \times 3) + 1$.