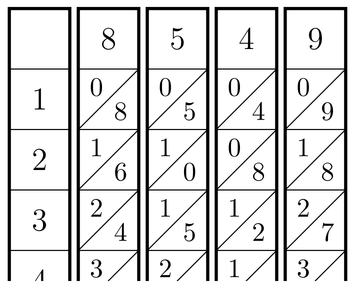
Week 2

The page you should have printed this week is a paper version of a device called Napier's Bones.

Multiplication

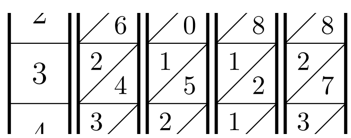
We'll start by multiplying **8549** × **3**. Follow these steps:

1. Start with the "1, 2, 3,. . . " bone, and arrange the other bones to its right so they spell "8549" along the top.



2. Read along row 3. Look at the diamond shapes formed by joining together triangles in adjacent bones. (At the start and end, there'll only be half-diamonds.)

3. In each diamond, add up the digits that appear.



2    $4+1$ $5+1$ $2+2$ 7

4. These sums are the digits of 8549 × 3.

Question 1. Did it work? Double-check the answer with a calculator.

Question 2. Try multiplying some more numbers:

(a) 273 × 5
(b) 846 × 8
(c) 395 × 6
(d) 68527 × 6
(e) 12345678 × 9
(f) 2222 × 3

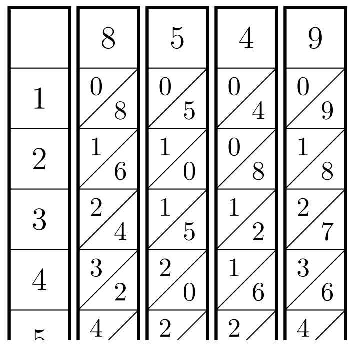
Question 3. Explain why this works. (What's the pattern in the numbers on the bones?)

Long multiplication

Napier's bones are great at multiplying things by single-digit numbers. But what if we want to multiply by a longer number? We can use Napier's bones to help us do *long multiplication*.

For example, let's multiply **8549** × **324**.

1. Arrange the bones so the top row spells out 8549, like before.



2. Use the bones to multiply 8549 by 3, 2 and 4.

$$8549 \times 3 = 25647, \qquad 8549 \times 2 = 17098, \qquad 8549 \times 4 = 34196$$

3. Write these numbers above each other, so that each number is shifted one place to the right from the one above it.

2	5	6	4	7		
	1	7	0	9	8	
		3	4	1	9	6

4. Add up the columns (and carry any excess tens digits over to the next column to the left).

2	5	6	4	7		
	1	7	0	9	8	
		3	4	1	9	6
2	7	6	9	8	7	6

5. The result is 8549 × 324.

Question 4. Try multiplying some other numbers:

(a) 71 × 62

(b) 854 × 38

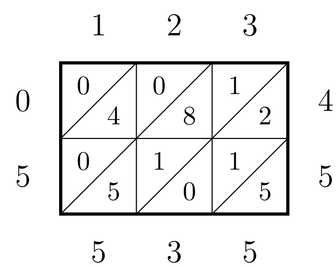
(c) 973526 × 2354

Question 5. Why does this work?

Venetian grids

Let's set Napier's Bones aside for a moment, and look at another technique to multiply numbers, this one only using pen and paper.

If I wanted to multiply **123** × **45**, for example, I would write this:



Question 6. What's going on in this technique?

Question 7. Use this technique to multiply:

(a) 67×89

(b) 286×7

(c) 394×551

(d) 1618×736

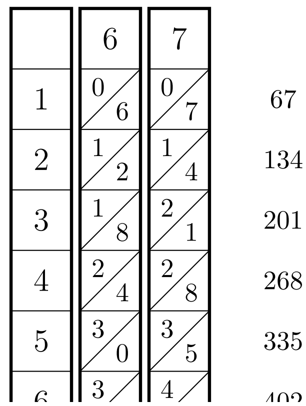
Question 8. Why does this technique work?

Division

We are now experts on multiplication! But what about division?

Let's calculate **22847 ÷ 67**.

1. Set up the bones to spell 67. Multiply 67 by all the single-digit numbers, and write down the answers.



2. Write this:

67	2	2	8	4	7

3. Take the first few digits of 22847 until we get a number bigger than 67 — we'll need the first three digits, 228. Find the largest multiple of 67 that's smaller than 228 — it's $3 \times 67 = 201$. Write 201 under 228, and write 3 above it.

			3		
67	2	2	8	4	7
	2	0	1		

4. Subtract 201 from 228 — we get 27. Bring down more digits from 22847 until we get a number bigger than 67.

			3		
67	2	2	8	4	7
	2	0	1		
		2	7	4	

5. Repeat steps 3 and 4 with this new number, 274, instead of 228.

			3	4	
67	2	2	8	4	7
	2	0	1		
		2	7	4	
		2	6	8	
				6	7

6. Keep going until we run out of digits in 22847.

			3	4	1
67	2	2	8	4	7
	2	0	1		
		2	7	4	
		2	6	8	
				6	7
				6	7
					0

7. The number in the top row, 341, is the answer.

Question 9. Divide some more numbers:

(a) $2730 \div 42$

(b) $41553 \div 513$

(c) $98153 \div 184$

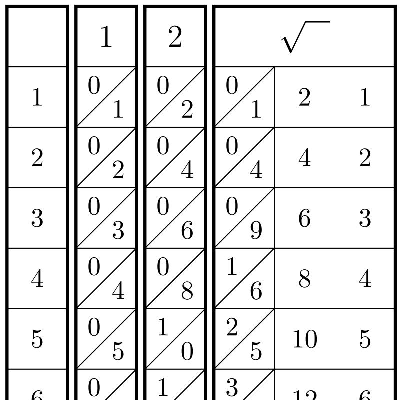
(d) $15943827 \div 6438$

Question 10. Why does this work?

Square roots

Napier's bones can do more than simple arithmetic! Let's calculate $\sqrt{\mathbf{413449}}$. We'll need the special square root bone.

1. Group the digits of 413449 into pairs: 41 34 49. Take the first pair, 41. Look at the leftmost column of the square root bone, and find the biggest number that's less than 41 — it's $36 = 6^2$. The number next to it on the square root bone is 12. Set up the bones and write the numbers like this:

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  6
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4 1 3 4 4 9
3 6
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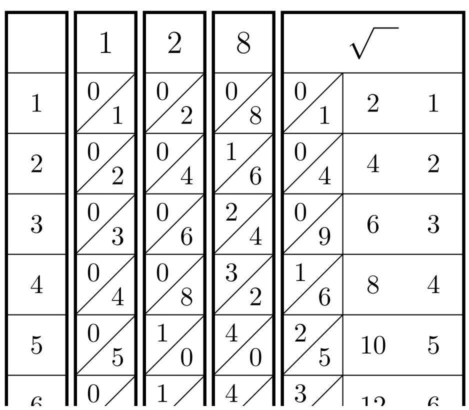
2. Subtract 36 from 41, to get 5. Pull down the next pair of digits in 41 34 49, to get 534. Read the rows of the bones like usual, *including the left column of the square root bone*, and find the biggest number that's less than 534 — it's row 4, which reads 496.

   In the middle column of the square root bone, in row 4, we find the number 8. The current bones spell 12: multiply this by 10 and add 8, to get 128. Reset the bones to spell 128.




```
  6   4
-----------
4 1 3 4 4 9
3 6
-----------
  5 3 4
  4 9 6
```

3. Repeat these steps until you're done:
    - subtract the numbers,
    - pull down the next pair of digits,
    - find the row that reads the largest thing less than this,
    - add the middle of this row of the square root bone to 10 times the current bones, and reset the bones.

	1	2	8	6	$\sqrt{\ }$		
1	0/1	0/2	0/8	0/6	0/1	2	1
2	0/2	0/4	1/6	1/2	0/4	4	2
3	0/3	0/6	2/4	1/8	0/9	6	3
4	0/4	0/8	3/2	2/4	1/6	8	4
5	0/5	1/0	4/0	3/0	2/5	10	5
6	0/	1/	4/	3/	3/	12	6

	6		4		3
4	1	3	4	4	9
3	6				
0	5	3	4		
	4	9	6		
		3	8	4	9
		3	8	4	9
					0

4. The top row of your table is the answer!

Question 11. Calculate some more square roots:

(a) $\sqrt{5776}$

(b) $\sqrt{152881}$

(c) $\sqrt{46656}$

(d) $\sqrt{258064}$

(e) $\sqrt{2329}$

(f) $\sqrt{7091835}$

Question 12. *(Challenge.)* Why the heck does this work?

Question 13. *(Even more challenge.)* Napier's Bones can be used to take cube roots too, using a special cube root bone. Design a cube root bone.