Week 1

Question 1. Jafar has four prisoners. He lines up three of them: Aladdin faces the wall, the Sultan stands behind Aladdin, and Abu stands behind the Sultan. The fourth prisoner, Jasmine, is put in a separate room. Jafar gives all four prisoners hats. He explains that there are two green hats and two blue hats, that each prisoner is wearing one of the hats, and that each of the prisoners in the line see only the hats in front of him/her. Jasmine can't see or be seen by any other prisoner. No communication among the prisoners is allowed.

If any prisoner can figure out what color hat he/she has on his/her own head with 100 % certainty (without guessing) and tell Jafar, all four prisoners go free. If any prisoner suggests an incorrect answer, all four prisoners are executed. Can the prisoners escape?

Question 2. Cruella de Vil lines up 101 dalmations in a single-file line and puts a blue or a pink hat on each dog's head. Every dog can see the hats of the dogs in front of him/her in the line - but not his/her own hat, nor those of anyone behind him/her. Cruella starts at the back of the line and asks the last dalmation the color of his/her hat. He/she must answer "blue" or "pink." If he/she answers correctly, he/she is allowed to live. If he/she gives the wrong answer, he/she is instantly and silently sent to the pound. On the night before the line-up, the dogs talk about a strategy to help them. What should they do?

Question 3. There are an even number of hyenas standing in a circle, and Scar is going to place a pink or blue hat on each of their heads. Each hyena can see everyone else's hat, but they do not know the total number of pink or blue hats. On Scar's command, each hyena will have to simultaneously say the color of their hat. If at least 50 % of the hyenas guess correctly, all hyenas will be set free. Can the hyenas devise a strategy to ensure they go free?

Question 4. There are 100 ruffians in the Snuggly Duckling with 100 boxes in a separate room, each containing the name of a ruffian (all boxes are unique). The ruffians take turns going into the room and open 50 boxes. The ruffians get to go free if they all open the box with their name in it, but otherwise they all get their dreams crushed. What strategy should they use?

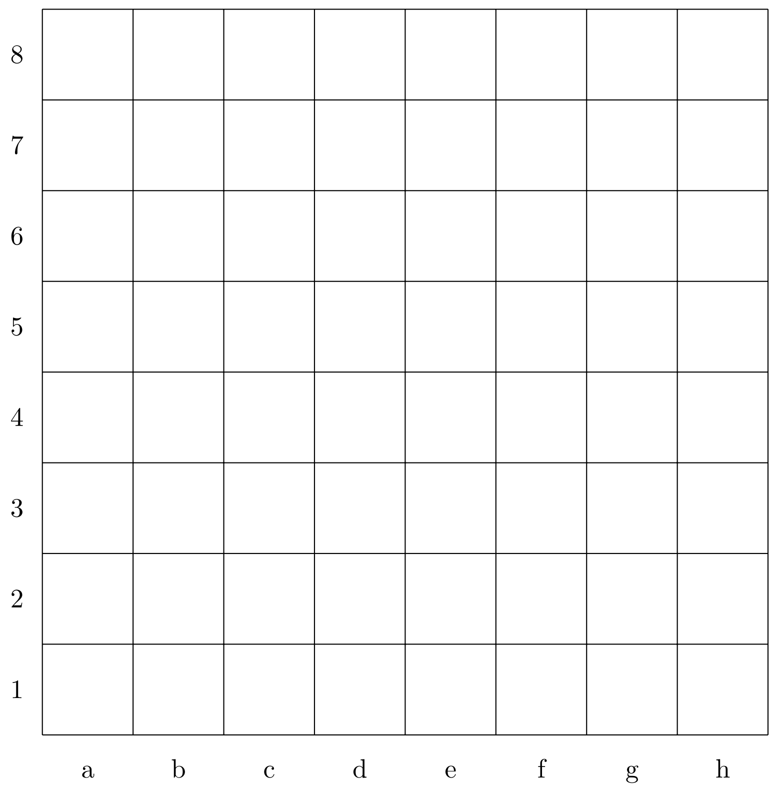
Question 5. Now there are again 101 dalmations in a line, and they have hats labeled 1 through 102 (one number is missing). Starting at the back of the line the dogs have to guess the number on their hat. If all the dalmations are correct they get to go free, and if anyone is wrong they all get sent to the pound. What strategy should they use?

Question 6. There are 100 Squeeze Toy Aliens in solitary cells. There is a central living room with one light bulb; this bulb is initially off. No alien can see the light bulb from his or her own cell. Everyday, Sid picks a alien equally at random, and that alien visits the living room. While there, the alien can toggle the bulb if he or she wishes. Also, the alien has the option of asserting that all 100 aliens have been to the living room by now. If this assertion is false, all 100 aliens are thrown in the trash. However, if it is true, all aliens are set free. The aliens are allowed to get together one night in the courtyard, to discuss a plan. What plan should they agree on, so that eventually, someone will make a correct assertion?

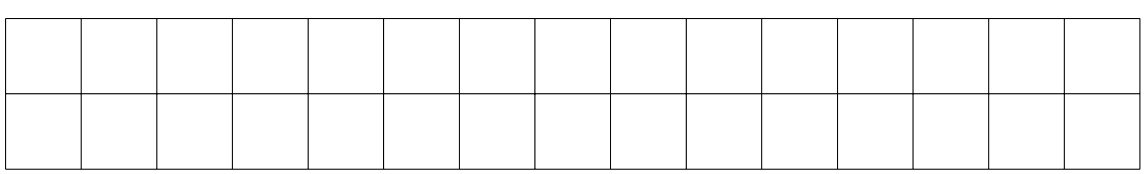
How long will your plan take? Is there a faster plan?

Question 7. (a) Play the following game on an 8×8 chessboard. Start with a queen on square b1. On your turn, you can move the queen any number of squares vertically, horizontally, or diagonally, but she must always move up and/or to the right. The winning player is the one who can move the queen to square h8. Who wins?

(b) What if the Queen starts on $a1$ or $c1$ instead?



Question 8. Timon and Pumbaa are playing a game on a 2×15 grid. They take turns tiling it with 1×2 dominoes, and the first person who can't place a domino loses. If Timon goes first, who wins? What if they used a 2×14 board instead?



Question 9. Ten ones and ten twos are written on the blackboard. Cri-Kee and Mushu play a game in which they take turns erasing numbers from the board two at a time. If a player erases two different numbers from the board, they replace them with a 1. If they erase two of the same number from the board, they replace it with a 2. Cri-Kee wins the game if the last number remaining on the board is a 1. Mushu wins if the last number left on the board is a 2. If Cri-Kee goes first, who wins the game?

Question 10. Who wins tic-tac-toe? If you already know that, who wins 3 dimensional tic-tac-toe? (In 3D tic-tac-toe, each player puts their 'X' or 'O' in any one of the 27 spaces in a $3 \times 3 \times 3$ cube. A player wins when they get three in a row vertically, horizontally, or diagonally).