

UW Math Circle  
February 20th, 2020

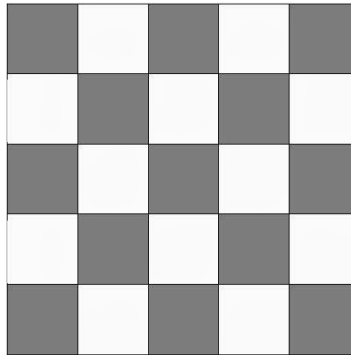
### Problems

1. If  $S$  is a set of positive integers, we can look at the *triple sum set* for  $S$ , given by all sums of three elements of  $S$ , i.e.

$$3S = \{r + s + t : r, s, t \in S\}$$

Find a set  $S$  of positive integers (0 not allowed) with the fewest number of elements possible such that  $\{1, 2, 3, \dots, 100\} \subset 3S$ .

2. Consider a 5 by 5 checkerboard. Place two white queens, and then place as many black queens as you can so that no queen can capture a queen of the opposite color. What is the largest number of black queens you can possibly place?



3. You have a coin that has probability  $3/7$  of being heads, and  $4/7$  of being tails. Design an algorithm to simulate the result of a fair coin using only flips of your biased coin, such that the average number of flips needed to output your result is as small as possible.
4. You can place dots on a sphere, and connect some of them by edges. What's the most dots and edges you can put down such that no two dots are more than two edges apart, no two edges cross each other, and every face of the resulting graph is a triangle?