UW Math Circle
October 17th, 2019

Digital Circuits - Homework 3

Problem 1: Rotate

Suppose there are 16 ordered input wires representing an integer in binary, $X = X_{15} \dots X_0$, and depending on a control wire S, one wants to build a circuit that optionally shifts them d places to the left, wrapping around - this is called a *rotation*. Specifically, if S is 0, then for each of the 16 output wires, the output wire $Y_i = X_i$, but if S is 1, then $Y_i = X_{(i-d) \mod 16}$. So, for example, if $n = 5$, then if S is off Y_2 is X_2, otherwise it is X_{13}.

(a) Using any gates we have encountered, build a circuit for the Y_i, that takes as input S, X_i, and $X_{(i-d) \mod 16}$.

(b) Since you now know how to build a circuit that rotates 16 bits by d places, let's call this gate $\boxed{\text{ROT}_{16}(d)}$. Using rotation gates, show how to build a circuit that takes as input the 16 bit binary integer X and a 4-bit binary representation of a number of places to rotate, $s = s_3 \dots s_0$, and outputs the shifted binary number Y. (That is, if $s = 0000$, then the circuit should output $Y = X$, and if $s = 0101$ then the circuit should output Y rotated 5 places.

Problem 2: Triangular truth

Using 2-input $\boxed{\text{OR}}$ and $\boxed{\text{AND}}$ gates, show how to construct a circuit that takes as input two n-bit inputs, $X_0 \dots X_{n-1}$ and $Y_0 \dots Y_{n-1}$, that outputs 1 if and only if there is some pair of input bits $X_i, Y_{j \geq i}$ that are both on. That is, if X_0 is on it should output 1 if any bit of Y is on, but if only bit 5 of X is on, it should output 1 only if some bit $Y_5, Y_6, \dots$ is on.

You may find that the number of gates in your circuit is approximately $2n^2$. Can you do this using only $2n$ gates?