UW Math Circle
October 10th, 2019

Warmups

Hungry lions

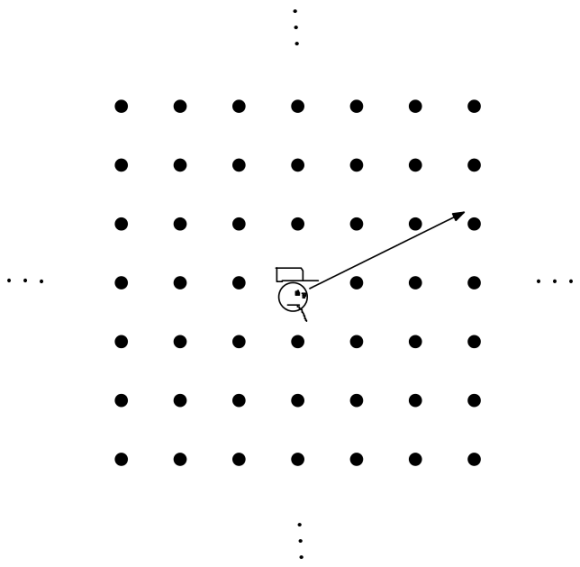
The island of Lionia has a population of 100 lions, and nothing else. The lions are all hungry; also, they are excellent logical thinkers. (Assume that lions prefer being alive and hungry to being dead.) The lions would try to eat each other, but since all the lions are equally tough, they would die in battle. However, a lion that has recently eaten is temporarily weak, and vulnerable to attack.

As it is, no lion wants to try any funny business. Suppose a single sheep is introduced to the island. Does the sheep get eaten? Why or why not? Does the number of lions matter?

Infinite apple orchard

In the apple dimension, there is an infinite two dimensional apple orchard. There is an apple tree at every lattice point – any point (x, y) where x and y are integers – except at $(0, 0)$, where the apple farmer is standing. Apple trees have no width – they are just points. How far can the farmer see in an any given direction? Which trees can the farmer see?

Suppose the trees are circular, with positive radius. How far can the farmer see now?



Nuts and bolts

The bolts shown have regular helical grooves. If you circle the bolts around each other in the directions indicated, in the way you would twiddle your thumbs, what will happen to the bolt heads? Will they move inwards, move outwards, or stay the same distance apart?