$\sim \heartsuit \sim$ *Valentine's Day Auction* $\sim \heartsuit \sim$

Problem 1. A *happy number* is a number that is:

- a square number,
- a triangular number,
- a prime number, or
- a palindrome.

A *happy couple* is a pair of consecutive numbers that are both happy. For example, $10, 11$ is a happy couple, because 10 is a triangular number ($10 = 1 + 2 + 3 + 4$) and 11 is a palindrome (because 11 backwards is still 11).

What's the largest happy couple you can find?

Problem 2. You want to grow some sugar cane to give to your crush. Sugar cane grows in $1\text{m} \times 1\text{m}$ blocks, but it will only grow if it's next to a $1\text{m} \times 1\text{m}$ block of water (horizontally or vertically, not diagonally). You have $10\text{m} \times 10\text{m}$ of space to grow the sugar cane. How do you arrange the sugar cane and water to get the most sugar?

Problem 3. The numbers from 1 to 100 are having a Valentine's Day ball, and they're performing the Euclidean Dance. In this dance, numbers form pairs, with the smaller number (called n) on the left and the bigger number (called N) on the right. In each step of the dance, the bigger number, N, turns into $N - n$. If n is still smaller than $N - n$, the dance continues; if n is now bigger than $N - n$, the numbers have to do-si-do to switch places. If n and $N - n$ are equal, the dance ends.

For example: the pair 5 and 12 becomes 5 and $12 - 5 = 7$, then they become 5 and 2, but now 2 is less than 5 so they do-si-do to become 2 and 5, and then 2 and 3, then 2 and 1 so they do-si-do again to become 1 and 2, then 1 and 1 and the dance ends.

What pair of numbers between 1 and 100 can you find that has to do-si-do as many times as possible?

Problem 4. You're writing a poem to your valentine. Your valentine is a topologist and therefore likes holes, so you want the words in your poem to have lots of holes in them. For example, the letter "o" has one hole, but "k" has none. Find a word with up to 10 letters that has as many holes as possible, when written in lower case.

Problem 5. Your secret admirer has given you a block of chocolate, in the shape of a 10×10 grid of squares. You can break the chocolate into pieces by snapping it along one of the grid lines. What order should you snap the pieces to break the chocolate into individual squares with the fewest number of snaps?