UW Math Circle
Worksheet

A coloring of a graph is a graph with every vertex a different color from each vertex it is connected to. We call the minimal number of colors needed for a coloring of a graph the chromatic number.

A bipartite graph is a graph where the vertices can be separated into two sets so that no two vertices in the same set share an edge.

1. Show that every bipartite graph has chromatic number 2.

2. Show that if a graph has chromatic number 2, it must be bipartite.

3. Find the chromatic number for K_n, the complete graph on n vertices.

4. A planar graph always has a vertex of degree 5 or less. Show that the chromatic number of a planar graph is at most 6. Can you show the chromatic number of a planar graph is at most 5?

5. Show that every planar graph must have a vertex of degree 5 or less.

6. Find an example of a planar graph with chromatic number 1. Can you find an example for chromatic number 2? For 3? For 4? For 5? For 6?

7. Consider the sudoku puzzle below. Can you solve it using graph coloring techniques? (Hint: Try solving it by hand first and think about what your vertices and edges should be).

		1	5					
8					3	1	4	
4			8					3
7			5					8
	6	9		8	3			
3			2			5		
5			7			1		
2	7	1				9		
				4	5			

8. A spanning tree for a connected graph G is a connected subgraph that contains all vertices of G with no cycles. Find spanning trees for K_1, K_2, K_3, K_4 and K_5. How many spanning trees can you find for each graph? What do you notice about the number of edges for each spanning tree?