UW Math Circle
Worksheet

1. Harad has n towns and $n - 2$ roads between towns. Show that you cannot travel from any town to any other town through a sequence of roads.

2. In Beleriand, there are $2n$ villages and every village has roads to at least n other villages. Show that you can travel from any village to any other village through a sequence of roads.

3. In Gondor, there are $2n$ towns, every town has roads to at least $n - 1$ towns, and the largest town has roads to at least n towns. Show that you can travel from any town to any other town through a sequence of roads.

4. A horde of orcs have attacked the town of Osgiliath in Gondor. Osgiliath no longer exists and all roads to Osgiliath no longer exist. Luckily, Osgiliath only had roads to $n - 1$ other towns. Show that you still can travel from any town to any other town through a sequence of roads.