

UW Math Circle

1. Nine people walk into a house and sit down at a table with six chairs so that everyone is sitting. How can we think of sitting as a function from the collection of nine people to the collection of chairs? Is the sitting function one-to-one, onto, both or neither?
2. What kind of functions can be constructed from the collection of all Volvos to the collection of all cars?¹
3. Suppose that there are three mice and four beds. If we think of sleeping as a function of mice to beds, how many different functions are there? How many of these functions are one-to-one, onto, both or neither?
What if there are three mice and three beds? Or three mice and two beds?
4. Given any two collections, are there any functions which will always exist between them?
5. What types of functions from the collection of vowels to the collection of letters exist?
6. Suppose that Collection A has n objects and Collection B has m objects. How many functions from Collection A to Collection B are there? How many of these functions are one-to-one?

¹Note: A Volvo is a type of car.

1. Here are some pairs of collections. For each pair, can you say which collection is bigger (even if you don't know the size of any of the collections)?

- Cars, or Red cars
- Cars, or Car manufacturing companies
- Cars that were made before 1990, or Cars that were made before 2010
- Yellow cars, or Purple cars
- Cars, or Car wheels
- Cars, or Car steering wheels
- Cars, or Sun roofs
- Cars, or Cup holders
- Cars, or Cars
- Cars, or Cats

What does this have to do with functions? For each pair, describe a function that helps you decide which collection is bigger (if you can).

1. The Fighting Soccer Slugs and the Serious Football Frogs are congratulating each other after their international soccer game. Each player shakes hands with exactly one player from the other team and all players have their hand shaken. Explain why shaking hands, as a function from one team to the other, is a correspondence. Can we say anything about the sizes of the teams?
2. The Slithering Snakes, a world-renowned gang of snake collectors, each have at least one pet snake. Show that there is an onto function from the collection of snakes owned by the Slithering Snakes to the members of the Slithering Snakes.
3. If there is an onto function of students from Cool Kid Elementary School to teachers, explain why there is a one-to-one function from teachers to students at Cool Kid Elementary School.
4. If there is a one-to-one function from students at Groovy Peeps Middle School to lunchboxes, explain why there is an onto function from lunchboxes to students.
5. Suppose you have a collection of 37 people, and a function from the collection to itself. If the function is one-to-one, explain why it must also be onto.
6. Suppose you have a collection of 37 people, and a function from the collection to itself. If the function is onto, does it have to be one-to-one as well?

7. Suppose you have a function from the collection of Aardvarks to the collection of Badgers, and another function from the collection of Badgers to the collection of Caterpillars. You compose the functions, and notice that the result is one-to-one. What can you say about the two functions you started with?

8. This time, you pick new functions from Aardvarks to Badgers and from Badgers to Caterpillars. Once again, you compose them, but this time you find that the result is onto. What can you say about the starting functions?

9. Suppose that the Dandy Dans take seats at a table so that each Dan gets a seat to themselves. When a whistle blows, all of the Dans run around the table and sit in a new seat. After the rearranging, every seat has at least one Dan sitting in it. What can you say about the collection of Dans and the collection of seats?

10. Suppose that the Fighting Soccer Slugs and the Serious Football Frogs are exchanging gifts. Every Fighting Soccer Slug gives exactly one gift to exactly one Serious Football Frog and every Serious Football Frog gives exactly one gift to exactly one Fighting Soccer Slug. Show that the function of gift giving is a correspondence.