

# UW Math Circle

## Week 1

Long ago, in a land decently far away, there was the Kingdom of Duloc. In Duloc, there was a castle full of enchanted doors. If you opened one of these doors, you might find a rotten onion or you might find a delicious cake.

Farquaad has put a sign on each door telling you what's inside, but not all of the signs are true!

1. When Shrek enters the castle, he encounters two enchanted doors. "One of the signs is true," says the Magic Mirror, "and the other is false."

I  
A delicious cake is behind this door. A rotten onion is behind the other one.

II  
A delicious cake is behind one of these doors. A rotten onion is behind the other.

Which door should Shrek open?

2. When Donkey enters the castle, he encounters two enchanted doors. "This time, either both signs are true or both signs are false," says the Magic Mirror.

I  
A delicious cake is behind at least one of these doors.

II  
A rotten onion is behind the other door.

Which door should Donkey open?

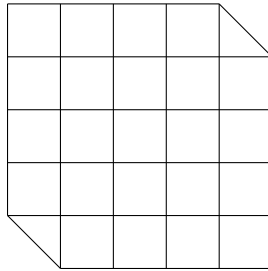
1. Farquaad tore out a section of successive pages from a book. The first page he tore out was numbered 165, and the last page he tore out was also numbered with the digits 1, 6, and 5 in some order. How many pages did Farquaad tear out of the book?
2. Suppose Dragon has 9 gold pieces, identical in appearance. You know one of them is fake and lighter than the others. If you have a balance scale, what is the smallest number of weighings it would take to find the fake coin?
3. The Seven Dwarfs found 700 gold coins in their mine. They decide to split the coins using this scheme: the oldest dwarf proposes how to share the coins, and ALL dwarfs (including the oldest) vote for or against it. If 50% or more of the dwarfs vote for it, then the coins will be shared that way. Otherwise, the dwarf proposing the scheme will be given nothing, and the process is repeated with the dwarfs that remain.

As dwarfs tend to be a greedy bunch, if a dwarf would get the same number of coins if he voted for *or* against a proposal, he will vote against so that the dwarf who proposed the plan will be given nothing.

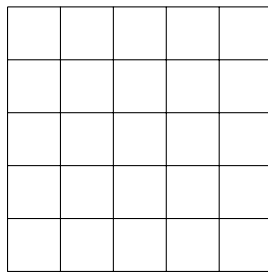
Assuming that all of the Seven Dwarfs are intelligent, rational and greedy, what is the maximum number of coins the oldest dwarf can get?



4. Using only one cut, can Princess Fiona cut the shape below into two pieces that can be moved to form a  $4 \times 6$  rectangle? (Fiona can only make one cut, but it can be a jagged line.)



5. Puss in Boots is trying to run from one corner of the neighborhood to another, but can't run on the ground because he's being chased by a dog! He is running along the tops of fences, which are arranged in a  $6 \times 6$  grid:



If he starts at the bottom left corner and wants to get to the top right corner and only runs up and to the right, how many different paths can he take?



6. How many ways can the Gingerbread Man cover a  $2 \times n$  chessboard with dominoes? (Assume each domino covers exactly  $2 \times 1$  or  $1 \times 2$  squares.)