UW Math Circle

Auction

1. The Consonant-Vowel sequence of a word counts the placement of consonants and vowels in a word. For example, "example" has Consonant-Vowel sequence "vcvcccv", where v stands for vowel and c stands for consonant. Find the longest pair of words with opposite Consonant-Vowel sequences. For example, (ash, boo) forms such a pair. We count the letter y as a consonant.

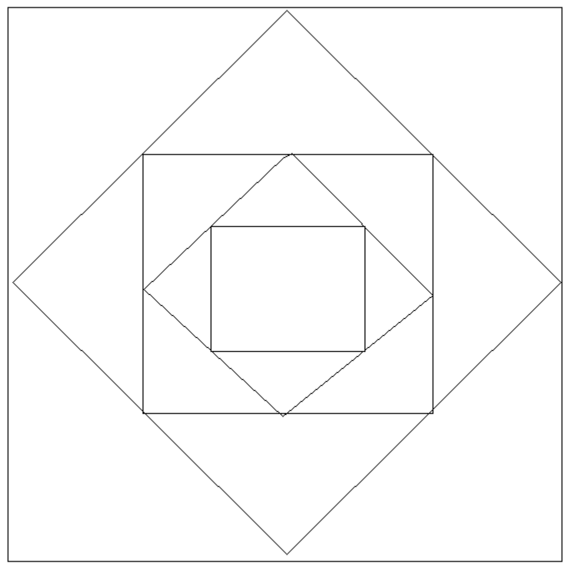
2. We define the love sign of a number to be the sum of all of its integer divisors, divided by the original number. For example, the love sign of 10 is $\frac{1+2+5+10}{10} = 1.8$ and the love sign of 9 is $\frac{1+3+9}{9} = \frac{13}{9}$. We say that two numbers are a match made in heaven if they have the same love sign. For example, (6,28) is a match made in heaven. Find two numbers that are a match made in heaven with the largest sum that you can.

3. Find an equation that uses each of the numbers 0,1,2,3,4,5,6,7,8,9 exactly once with the largest value on both sides of the equation. For example,

$$536^{18/4} \cdot (9 - 7 - 2) = 0$$

is an equation with value 0.

4. An example of a square to overcome our differences is below. You'll see that in each square, the midpoint is the positive differences of the numbers at the corners. We say such a square is successful when the inside square has 0 on each corner. We count the layers of successful squares by counting the number of squares contained in it. The example below has 5 layers. Find a successful square to overcome our differences with the most layers that you can.



5. A number is popular if it is the product of a perfect square and perfect cube. For example, $25 = 25 \cdot 1, 27 = 1 \cdot 27, 32 = 4 \cdot 8, 72 = 9 \cdot 8$ are all popular numbers. We say that two numbers form an "it couple" if they are consecutive popular numbers. Find an it couple with the largest sum that you can.

6. Find the longest sequence of primes less than 100 such that each prime is a proper factor of the product of all previous primes plus one. For example, the sequence (5,3,2) is valid because 3 is a proper factor of $5+1$ and 2 is a proper factor of $5 \cdot 3+1$ but (5,3,2,31) is not valid because 31 is not a proper factor of $5 \cdot 3 \cdot 2 + 1$.