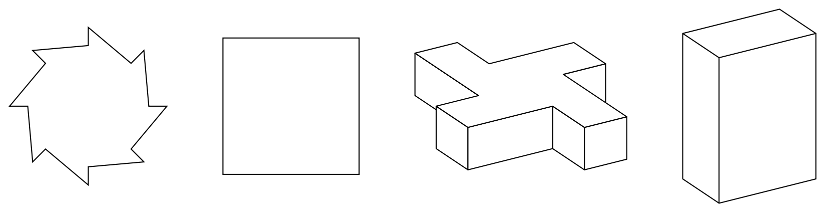
Problem 1. Jabba the Hutt collects mattresses. Here are some of his favourites! The first two are 2D, and the last two are 3D:



Jabba has a Reflexor Engine that can send a 3D mattress to a warp dimension and return a mirror-image version.

What many mattress-flipping actions are there for each mattress, if you're allowed to use the Reflexor as part of the action? How many actions are there for each?

Problem 2. Write down permutation notation for the actions.

Problem 3. These four mattresses all have the same number of flips, but the symmetries are still different. Let's try to find some attributes that distinguish them!

(a) For each mattress, what's the smallest list of actions that you can use, to get every possible action by doing actions on your list in some order?

(b) Some mattress actions can be done in any order. For example, if you rotate by 90° then rotate by 180°, the result is the same as rotating by 180° first then by 90°.

Does Jabba have a mattress where all the actions can be done in any order?

(c) The “order” of a mattress action is the number of times you have to do it before you get back to where you started. For example, the order of rotating by 90° is 4, since doing this action four times has the same result as doing nothing. Which mattresses have an action of order 4? Order 8? Order 2? Order 3?

(d) For every mattress action, there is an action that un-does it and brings you back to what you started with — this is called its “inverse”. Sometimes this is a different action than the first one, but sometimes it’s the same.

Which actions are their own inverses? Does this have something to do with the previous question?

Does the order matter when you do an action and its inverse?

The sets of mattress symmetries we've been looking at are an example of something that mathematicians like studying, called "groups".

A *group* is a set of things (e.g. mattress actions) with a way of combining them (e.g. "do this action first, then that one"), satisfying the following rules:

- There's a special thing in the set (e.g. the "do nothing" action) where combining it with anything else in the set doesn't change the other thing — for example, rotating by 90° and then doing nothing is the same as just rotating by 90°. This special thing is called the "identity element".
- Every thing in the set has an inverse thing, meaning that if you combine the thing and its inverse, you get the identity element.
- The way of combining things is "associative". (This is a technicality that you can usually ignore.)

Problem 4. Check that these sets are groups. In each case, what is the special identity element?

(a) Integers, where the way of combining things is adding them

(b) Permutations, with the way of combining them that we talked about two weeks ago

(c) Integers modulo 7, with adding

Problem 5. Which of these sets are groups? (If they aren't, can you fix them?)

(a) Real numbers, where the way of combining is multiplication

(b) Integers modulo 7, with multiplication

(c) Integers modulo 8, with multiplication

Problem 6. Which of the groups in problems 4 and 5 have which properties from problem 3?