

## UW Math Circle

Boba Fett has entered a mattress flipping competition. In the competition, every competitor has to pick a set of moves that lets them put the mattress in every sleepable position by doing some of their moves in some order. A competitor wins if they are able to put the mattress into every sleepable position and they have the smallest number of moves needed. For each of the shapes below, determine a winning set of moves for Boba Fett.

Wicket Wystri Warrick has noticed that the possible sleepable positions of mattresses are related to the symmetry of the mattress. What is this connection? Can we say something about the needed number of distinct moves?

Exogorth has a mattress shaped like a hollow rectangular prism which he sleeps inside of. If Boba Fett tries to flip this mattress, will he need a different strategy from the rectangular mattress? If so, what should he do?

In the autumn, we learned about permutations of numbers and cycle notation. For each of the permutations described below, the corresponding cycle notation is written below it:

$$1 \mapsto 5, \quad 2 \mapsto 3, \quad 3 \mapsto 2, \quad 4 \mapsto 4, \quad 5 \mapsto 6, \quad 6 \mapsto 1$$
$$(156)(23)(4)$$

$$1 \mapsto 2, \quad 2 \mapsto 3, \quad 3 \mapsto 1$$
$$(123)$$

$$1 \mapsto 4, \quad 2 \mapsto 3, \quad 3 \mapsto 2, \quad 4 \mapsto 1,$$
$$(14)(23)$$

$$1 \mapsto 1, \quad 2 \mapsto 2, \quad 3 \mapsto 3, \quad 4 \mapsto 4, \quad 5 \mapsto 5$$
$$(1)(2)(3)(4)(5)$$

Using permutations, try to describe how to flip a rectangular mattress into a new sleepable position. Then try for all the other shapes!