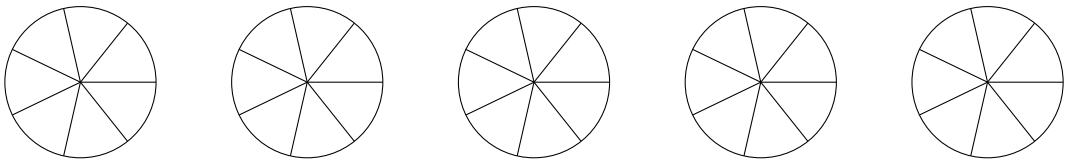
Wolstenholme

Ollen's Meowth has a bank vault with 5 dials, each of which has 7 segments.



Ollen's Meowth wants to color the dials so that there are 21 blue segments and 14 red segments. Write a formula for the number N of such dial colorings.

Ollen's Meowth splits up the N dial colorings into groups which can be reached from one another by rotating the dials. Show that the number of dial colorings in each group is a power of 7.

How many dial colorings are in a group of size 1?

How many dial colorings are in a group of size 7?

Show that $N \equiv 10 \pmod{49}$.