UW Math Circle
Week 6 Homework

1. A graph is called planar when it can be drawn so that no two edges intersect. Draw some examples of planar graphs.

   The dual of a planar graph G is a graph H that has a vertex for each face of G and an edge whenever two faces of G are separated by an edge. For each of your examples above, draw the dual graph. Do you notice anything special about the dual graphs?

2. A cycle in a graph is a collection of edges and vertices where each vertex is reachable from itself. Examples of graphs with cycles are below.

   The graph below is called the Petersen graph.

   What is the biggest cycle you can find? What is the smallest cycle you can find? Can you draw the graph again with the smallest number of edges crossing another edge?

3. The complete graph on n vertices, K_n, is a graph such that every pair of vertices is connected by exactly one edge. Some examples are drawn below.

Prove that

$$\binom{n}{2} = \binom{k}{2} + n(n-k) + \binom{n-k}{2}$$

using complete graphs and counting arguments.