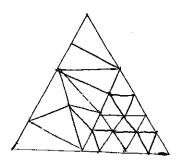
## UW Math Circle Week 4

1. How many ways can a regular n-gon be divided into n-2 triangles if n=3? What if n=4? What if n=5? What if n=6? Count different orientations as separate ways!

2. Grima is building a staircase of length 4 and height 4. How many ways can be build the stair case if he can only use 4 rectangles? The example for a staircase of height and length 2 is given below.



3. Color every vertex of the triangle below so that: the top vertex is red, the bottom left vertex is green, the bottom right vertex is blue, any vertex on the left side is red or green, any vertex on the right side is green or blue and any vertex on the bottom is red or blue. The vertices inside the triangle can be any color. Can you find a coloring so that there is no triangle with one vertex of each color?



4. How can you use Pascal's Triangle to calculate the powers of 11?

5. Suppose there are 100 light bulbs with light switches in a hall, numbered 1 through 100, that are all turned off. As Theoden walks down the hall, he flips all switches that are multiples of 1, then flips all switches that are multiples of 2, then flips all switches that are multiples of 3, and so on until he flips all switches that are multiples of 100. What light bulbs are on after Theoden completes his walk?

6. Aragorn draws 5 dots on a page and connects them with lines so that the lines do not cross each other as they move from dot to dot and every dot is connected to every other by a series of lines. He counts up the number of dots (D), lines (L) and regions (R) and then calculates D-L+R. What number did he get? Is there more than one answer? What if he started with 6 dots? Or 10? Or 100?

7. How many odd numbers are in the *n*-th row of Pascal's Triangle?

8. How many people must Saruman invite to a dinner party to ensure that three of his guests all know each other or are all strangers?