UW Math Circle (Cardinality)

1 Definitions

A *set function*, $f : S \to T$, is a rule that given an element of the set S, outputs an element in the set T. A set function is *injective* if whenever $f(x) = f(y)$ then $x = y$. A set function is *surjective* if for every element t of T there exists an element s of S so that $f(s) = t$. A set function is *bijective* if it is both surjective and injective.

1. Give an example of a set that is
   - injective but not surjective;
   - surjective but not injective;
   - bijective;
   - neither injective nor surjective.
2. Here is another definition for a bijective set function: The function $f : S \to T$ is bijective if there exists another function $g : T \to S$ such that $f(g(t)) = t$ for all t in T and $g(f(s)) = s$ for all s in S. Prove that this definition agrees with the one given above.

We say that two sets have the same *size* if there exists a bijection between them. We will use $|S|$ to denote the size of the set S. If there is an injection $f : S \to T$ then we say that $|S| \leq |T|$. If there is a surjection $f : S \to T$ we say that $|S| \geq |T|$.

2 Bijections

1. Find a bijection between the set of points on a circle and the set of points on a line.
2. Find a bijection between the the intervals $(0, \infty)$ and $(0, 1)$.
3. Is there a bijection between $(0, 1)$ and $(0, 1]$?

3 Power sets

The *power set* of a set S is the set of all subsets of S.

What is the power set of the set $S = \{a, b, c\}$? How many elements does it have? What about $T = \{0, 1, 2, 3\}$

4 Higher Cardinalities

Give an example of a set that people actually use elements of, that has cardinality greater than the real numbers.

5 Loose ends

If there exists an injection $f : S \to T$ and another injection g :