UW Math Circle (Voting Thoery)

November 10th, 2016

1 Ways to Vote

1. The points method: Every time that a candidate wins first place on someone's ballot, give them 3 points. Every time they win second, give them 2 point. Every time they win last, give them 1 point. Total the points and determine the winner. By this method, white chocolate is the winner. (In class we also proved that this method is equivalent to giving 1 point for first place, 0 points for second, and -1 point for third place).

2. The first-place method: whichever candidate won first place the largest number of times is the winner. By this method, milk chocolate wins.

3. Monarchy: One person (during class this was Enzo) is the monarch. Only the monarch's vote counts. Enzo happened to like dark over milk over white, so the winner was dark chocolate.

4. Elimination by fewest number of first places: Find the candidate that had the fewest number of first place votes (in this case, dark chocolate). Ignore that candidate. Now white chocolate has four more first place votes and milk has three more first place votes and they both total to 13. A tie!

5. Elimination by largest number of last place votes: Find the candidate that had the largest number of last place votes (in this case, dark chocolate) and ignore that candidate. In this case, we happen to eliminate the same candidate as in the previous method, so we again have a tie.

6. Elimination by points: Find the candidate that had fewest number of points (calculated by the points method above) and remove this candidate, shifting up all of the candidates below them. By this method, white chocolate wins.

7. Brackets: Pick a pair of candidates. Whoever wins between these two candidates then competes with the third candidate (and the loser is eliminated). Whoever wins between these two then competes with the fifth, etc. The winner is the last remaining candidate (the one who was never eliminated).

2 Voting Criterion

1. 1-1 winner: If there exists a candidate that beat every other candidate in a 1 on 1 race (ie, if we consider only these two candidates and count the number of times that one beats the other), then this candidate should win.

2. 1-1 loser: If there exists a candidate that lost to every other candidate in a 1 on 1 race, they should not win.

3. Not "meh": If a candidate was nobody's first choice, then this candidate should not be the winner.

4. Not backwards: Suppose that we counted up the votes (using one of the approaches we identified in the previous meeting) and we found that candidate A won. Then, suppose some voters changed their vote and ranked A higher than they had before. We want to make sure sure that in our voting system this does not hurt A - A should still be the winner after more people voted for her.

5. No ties: we want our voting system to give a definite winner.

6. Independent of Irrelevant Alternatives (IIA): Suppose that we counted up the votes and found that A won. Then suppose that some voters changed their votes but no one changed their mind about B relative to A. Then B should not win.

7. Unanimity: If every voter ranked candidate A over candidate B, then B should not win.