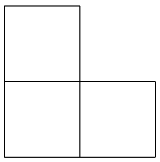
UW Math Circle

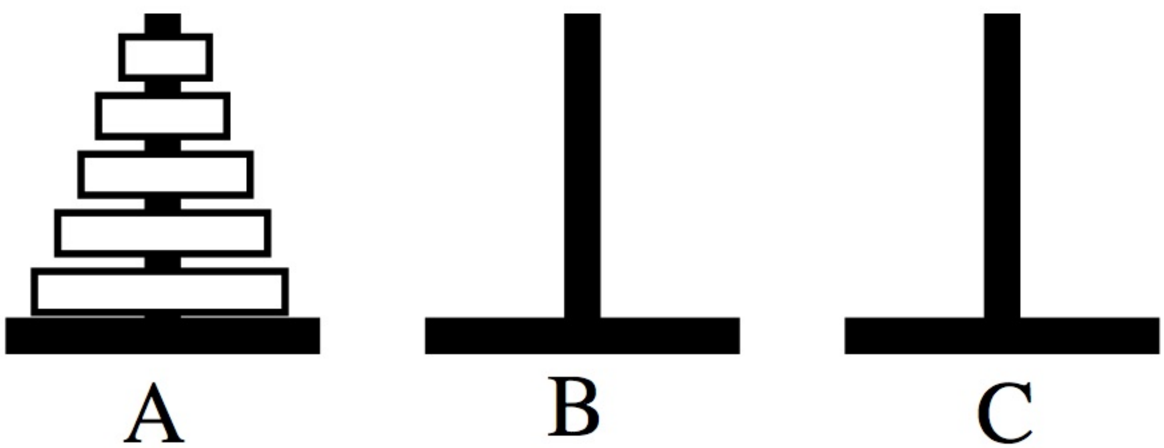
March 2, 2017

First, we'll start with the problems done in class. You showed that:

1. A tromino is an L-shaped piece, drawn below. It possible to cover a $2^n \times 2^n$ chessboard with the upper left corner removed with trominoes. We did this by showing it was possible for small n, and then "gluing" together 4 $2^{n-1} \times 2^{n-1}$ boards plus one extra tromino to cover the $2^n \times 2^n$ board.



2. In the game *'The Towers of Hanoi'*, there are three spindles on a base, with n rings on one of them. The rings are arranged in order of their size - from largest on the bottom to smallest on the top. You can move the highest (smallest) ring on any spindle onto another spindle, but you cannot put a larger ring on top of a smaller one. We showed that it was possible to move all of the rings to one of the other spindles. We did this by showing it was possible for small values of n, and then showed that oncee you knew how to do it for $n-1$ rings, you could do it for n rings: first, move the top $n-1$ rings to a free spindle. Second, move the bottom ring to the other spindle, and finally, move the $n-1$ top rings to that spindle.



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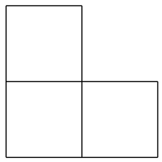
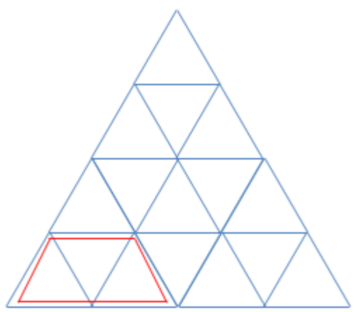
Using induction and the ideas from those problems (building up the n^{th} case from the $n-1^{st}$) case, try to do the following problems.

1. Is it possible to cover a $2^n \times 2^n$ chessboard with **any** piece removed with trominoes?



2. Take an equilateral triangle and cut it into 4^n congruent equilateral triangles. (This happens if you "cut" the triangle into 2^n rows along each side.) The $n = 2$ case is pictured below. If you remove the top triangle, prove that it is possible to cover the resulting shape with trapezoids, as drawn in the picture below.



3. In the game *'The Towers of Hanoi'*, prove that you can move all of the n rings to a free spindle using $2^n - 1$ moves, and it is not possible to do it with fewer moves.

4. How many edges are there in the complete graph with n vertices? (Remember, the complete graph is the one where you draw n vertices and connect each one to every other vertex. We did this problem before, but try to do it with induction!)