UW Math Circle

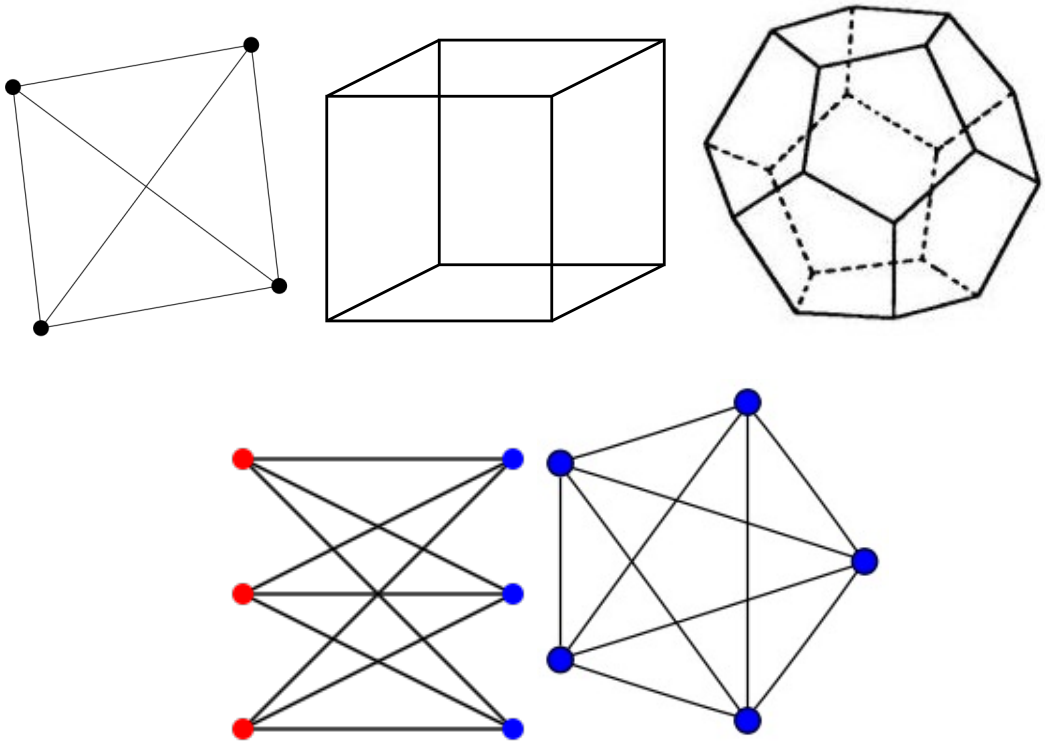
February 23, 2017

1. We call a graph **planar** if we can draw it in the plane without any of the edges crossing. A *face* of a planar graph is a region bounded by the edges. Note that the region outside a graph is also a face.

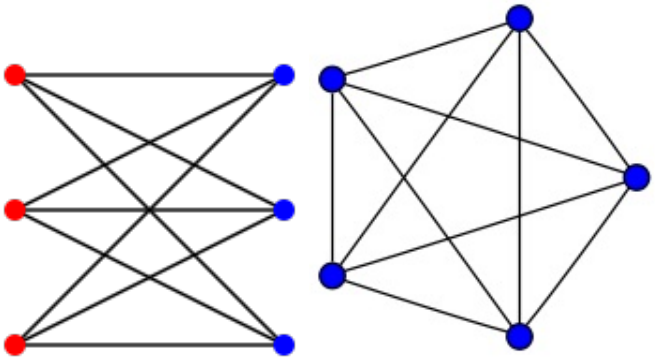
   (a) What is the minimum number of edges bounding a face for a graph with more than three edges and without multiple edges between two vertices?

   (b) For a planar graphs without multiple edges between vertices and with more than 3 edges, show that 2# of edges $\geq$ 3# of faces.

2. Which of the following graphs are planar? For each planar drawing that you find, find #vertices $-$ #edges $+$ #faces.



3. (a) You proved in problem 1 that $2E \geq 3F$ for planar graphs. Did that formula help you show that these graphs aren't planar?



(b) Together, we proved that, for a planar graph, $V - E + F = 2$. Can you use this and the formula $2E \geq 3F$ to show that those graphs aren't planar?

(c) Show that if a graph contains no triangles (paths of length three that start and end at the same vertex), then $2E \geq 4F$. Can you use this and the formula $V - E + F = 2$ to show those graphs aren't planar?

4. For a planar graph, use Euler's formula to show that $E \leq 3V - 6$.

5. For a planar graph without multiple edges between two vertices, show that the average degree of a vertex (the number of edges coming out of it) is less than 6.