UW Math Circle
February 23, 2017
Homework

1. In the land of Tripleville, there are 3 roads leading in and out of each city. Is it possible for there to be 99 cities in Tripleville? What about 100 cities?

2. In the neighboring land of Centropolis, 100 roads lead out of each city, and it's possible to travel along these roads from any city to any other. One of the roads is closed for repairs. Prove that it's still possible to get from any city to any other city.

3. (a) Find a planar graph whose vertices cannot be colored with three colors so that no two adjacent vertices share a color.

    (b) From the worksheet, we know that a planar graph without multiple edges between vertices has a vertex of degree less than 5. Use this to show that every planar graph can have its vertices colored with six colors, where two vertices have different colors if they share an edge.