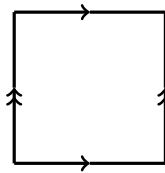


UW Math Circle
February 2, 2017
Homework

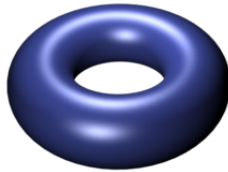
In class we talked about how to represent surfaces by drawing rectangles and gluing some of the edges together. Some of the gluings we can't do in three dimensional space, but we can still talk about the resulting surface! Remember, the arrows tell us which edges are identified, and what twists we are supposed to put in.

We can understand what happens when we cut surfaces and paste them together along their edges by understanding what happens to the rectangles when we cut and paste them together, but remember that all the cutting and pasting you do has to make sense with how the edges are identified.

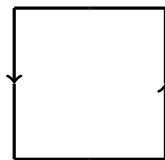
For example this:



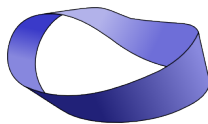
represents this:



While this:



represents this:

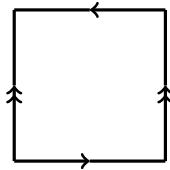


Use what you learned in class to answer the following questions!

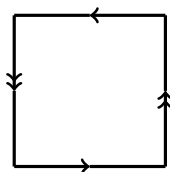
1. In class we explored what happens when we cut möbius bands.

Make a möbius band with one twist in it that is thick enough that you'll be able to cut it down the middle several times.

- (a) Cut it down the middle once. How many sides and twists does the resulting surface have? What does the shape of the möbius band look like, compared to what you had before?
 - (b) Compare the shape of the boundary of the möbius band before you cut it to the shape of the center of the möbius band after you cut it.
 - (c) Now cut it again down the middle. What does it look like now?
 - (d) Make a möbius band with three twists in it. Look at the boundary of the band. What happens when you cut the band down the middle?
2. Consider the surface given by this diagram:



- (a) How many sides does this surface have?
 - (b) What happens when you cut this surface vertically down the middle?
 - (c) What happens when you cut this surface horizontally across the middle?
 - (d) What happens when you cut a vertical rectangle out of this surface?
 - (e) What happens when you cut a horizontal rectangle out of this surface?
3. Consider the surface given by this diagram:



Joe lives on this surface, and he has a rope he wants to tie around the surface. He does this by looping it once across the middle and tying the ends together. He pulls on his rope, and it doesn't go anywhere.

He then decides to make it even stronger by looping it twice across the middle and then tying the ends together. He again pulls on his rope, and without the rope breaking he is able to pull all of the rope to where he is standing. Explain how this could have happened! It may be helpful to draw the rope on this surface in various places as Joe is pulling on it.