# UW Math Circle

January 19th, 2017

Homework

1. What is the last digit of $1^2 + 2^2 + 3^2 + \cdots + 9^2$? What about $1^2 + 2^2 + 3^2 + \cdots + 99^2$?

2. Show that $855$ cannot be written as $a^2 + b^2$ where $a$ and $b$ are integers.

3. Let $a$, $b$, and $c$ be the side lengths of a right triangle, and suppose they are all integers. Show that at least one of $a, b$, and $c$ is divisible by 3, and at least one of $a, b$, and $c$ is divisible by 5.

4. On a tropical island, far far away, there is a chameleon colony that lives by the following rules: the chameleons are three colors–either red, yellow, or blue–and whenever two chameleons of opposite color meet, they both change to the third color. So, if a red and yellow chameleon meet, they both become blue. If the island starts with 10 red chameleons, 11 yellow chameleons, and 12 blue chameleons, is it possible for them to eventually become all the same color?