

UW Math Circle
January 12th, 2016
Homework

In class, we continued talking about the pigeonhole principle. It will remain useful for some of these problems!

The Pigeonhole Principle. If $N+1$ pigeons fly into N birdhouses, at least one birdhouse will have at least two pigeons in it.

1. You are painting a large wall with two colors. Show that no matter how you paint the wall there will be two points one foot apart that are the same color (each point on the wall is painted a unique color).
2. You have a 3×3 grid, and in each square you write either $-1, 0$ or 1 . Show that now matter how you label the grid at least two of the rows, columns, or diagonals of the grid give the same number when you add up the numbers in the row, column, or diagonal.

For example, in the grid below the first column and the second column both sum to -1 .

-1	-1	-1
0	-1	1
0	1	1

3. The number 3 evenly divides 15 because $15 = 3 \cdot 5$. The number 15 is one less than a power of two because $15 = 2^4 - 1$. The number 7 evenly divides 63 because $63 = 7 \cdot 9$. The number 63 is also one less than a power of 2 because $63 = 2^6 - 1$.
 - (a) Show that 2017 evenly divides one less than a power of two, i.e. there is some some positive number M so that 2017 evenly divides $2^M - 1$. (**Hint:** Some of the ideas in problem four from the worksheet might be helpful)
 - (b) What numbers have the property that they evenly divide one less than a positive power of 2?
4. You have a 2016×2016 grid, and it is colored with 2017 different colors. You are allowed to recolor the grid in the following way:

If a row contains two identical colors, you can recolor the row that color (so that every square in that row will now be that color). Also, if a column contains two identical colors, you can recolor that column that color (so that every square in that column will now be that color).

Can you recolor the grid so that it is one solid color?