UW Math Circle

March 16, 2017

Auction

1. Find a sentence with **as many words as possible** so that the length of the words *strictly* increases through the sentence. For example "A man eats apples." is an example of such a sentence, while "A woman eats apples." is not (because "woman" has more letters than "eats").

2. Austin has an infinite plane that he wants to paint. He wishes to paint it in such a way so that points that are 1 foot apart have different colors, and he wants to do so using as few colors as possible (painting the plane means that he is going to assign a color to each point).

   (a) Find a **lower bound** for the number of colors that Austin must use that is as good as possible. (i.e. find as big as possible an n so that you can prove that Austin can't achieve this coloring with n colors).

   (b) Find an **upper bound** for the number of colors that Austin must use that is as good as possible. (i.e. find as small as possible an n so that you can find a coloring of the plane satisfying the desired property with n colors).

3. The totient of a natural number is the number of natural numbers less than that number and relatively prime to it. We denote the totient of n by $\phi(n)$. For example, $\phi(1) = 0$, $\phi(2) = 1$, $\phi(3) = 2$, and $\phi(4) = 2$ ($\phi(4) = 2$ because although there are three natural number less than 4, the number 2 is not relatively prime to 4).

   Your task is the following: **find as large a set as possible of numbers from** 1 **to** 1000 **so that all the numbers in your set have the same totient**.

4. Given a number there is a sequence starting with that number where each subsequent number is the sum of the divisors of the previous number. For example, starting with 12: the divisors of 12 are $1, 2, 3, 4, 6$. The sum of these is 16. The divisors of 16 are $1, 2, 4, 8$. The sum of these is 15. The divisors of 15 are $1, 3, 5$. The sum of these is 9. The divisors of 9 are $1, 3$, whose sum is 4. The sum of the divisors of 4 is 3, and the sum of the divisors of 3 is 1. So, the sequence goes: $12, 16, 15, 9, 4, 3, 1$.

   Your task is: **find a number between** 1 **and** 130 **where the sequence starting with your number is as long as possible**.

5. Place **as many dots as possible** on the gridpoints of the attached sheet of paper so that no five of the dots form a convex pentagon and no three of the dots are collinear.

6. In the puzzle below, you can slide any square into the empty square. You want to move the squares so that the numbers $1 - 8$ are in order and so that the blank square is at the end. What is the **smallest number of moves** you can solve the puzzle in?

   Record your sequence of moves by drawing the puzzles sequentially on graph paper. You don't need to include all the numbers each time, just include where the blank square is and what square you have moved.

Starting configuration:

	6	1
4	7	8
5	3	2

Goal configuration:

1	2	3
4	5	6
7	8	