

# UW Math Circle

March 9, 2017

1. Show that  $1 + 3 + 5 + \cdots + 2n - 1 = n^2$ .
2. Show that if there are several lines dividing the plane into different regions you can always color the regions black and white so that regions which share a border are different colors.
3. Remember: the Fibonacci numbers are the sequence  $1, 1, 2, 3, 5, \dots$ , where the  $n^{\text{th}}$  Fibonacci number is the sum of the previous two. In symbols:  $F_n = F_{n-2} + F_{n-1}$ .
  - (a) Find an expression for  $F_1 + F_2 + F_3 + \cdots + F_n$ , and use induction to prove that your expression is correct.
  - (b) Find an expression for  $F_1 + F_3 + F_5 + \cdots + F_{2n-1}$ , and prove that your expression is correct.
  - (c) Find an expression for  $F_2 + F_4 + F_6 + \cdots + F_{2n}$ , and prove that your expression is correct.
4. Show that  $1^2 + 2^2 + \cdots + n^2 = \frac{(n)(n+1)(2n+1)}{6}$ .
5. Show that  $111\dots 111$  (there are  $3^n$  ones) is divisible by  $3^n$ .