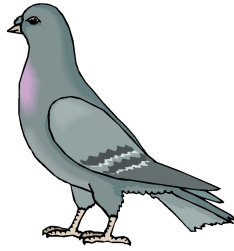


UW Math Circle
January 5th, 2016
Homework



In class, we talked about the pigeonhole principle. It will be useful for the following problems!

The Pigeonhole Principle. If $N+1$ pigeons fly into N birdhouses, at least one birdhouse will have at least two pigeons in it.

The Generalized Pigeonhole Principle. If $kN + 1$ pigeons fly into N birdhouses, at least one birdhouse will have at least $k + 1$ pigeons in it.

1. Show that an equilateral triangle cannot be completely covered by two smaller equilateral triangles.
2. Show that in any group of 5 people, there must be at least two people that have the same number of friends among those 5 people.
3. There are 100 people seated at a round table (spaced an equal distance apart from one another) and more than half of them are women. Show that there are at least two women seated directly across from one another.
4. You have 8 distinct positive numbers that are no greater than 15. Show that at least three pairs of them have the same positive difference (the positive difference of two numbers is the difference that is positive, for example the positive difference of 3 and 7 is 4). The pairs don't have to be non overlapping, for example I could have 5 and 7, 3 and 5, and 1 and 3 as 3 pairs with positive difference 2.