UW Math Circle

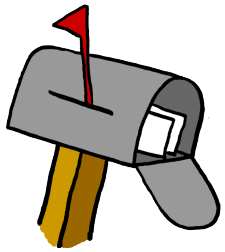
December 1, 2016

Two problems are ones from two weeks ago we didn't have much time to talk about.

1. There are 4 boys and 6 girls on a basketball team. How many ways can you choose 5 team members to be on the court, if there must be a girl and a boy on the court?

2. You have 20 sheep, 20 cows, and 20 goats, and each animal is indistinguishable from others of its species. In how many ways can pick 20 animals out of these 60?

3. A mail carrier delivers mail to the nineteen houses on the east side of Elm Street. The carrier notices that no two adjacent houses ever get mail on the same day, but that there are never more than two houses in a row that get no mail on the same day. How many different patterns of mail delivery are possible?



4. 16 people are seated at a round table. How many ways are there for each person to shake hands with one other person, so that no one's arms cross anyone else's?

5. You have n pairs of parentheses, and you want to count how many ways you can arrange these parentheses and have a valid arrangement of parentheses.

   For example if $n = 2$ you have parentheses $(, (,),)$, and the valid arrangements are: ()() and (()). Something like ())(or))((isn't valid: in ())(the third parentheses is a closed parentheses, but there is no open parentheses that it is closing. In))((, the first parentheses is a closed parentheses, which is invalid.

   How many valid arrangements are there with 10 pairs of parentheses? Can you relate this to a previous problem?

6. (a) How many ways are there to toss a coin 1 time so that there aren't 3 heads in a row? How many ways are there to toss a coin 2 times so that there aren't 3 heads in a row? How many ways are there to toss a coin 3 times so that there aren't 3 heads in a row?

   (b) Now, how many ways are there to toss a coins 4 times so that there aren't 3 heads in row?

   (c) How many ways are there to toss a coin 12 times so that there aren't 3 heads in a row?