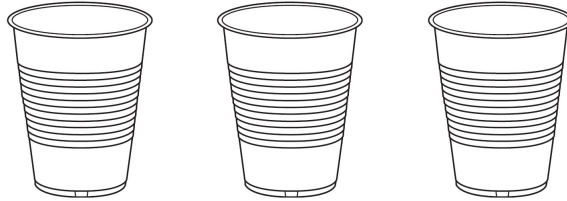


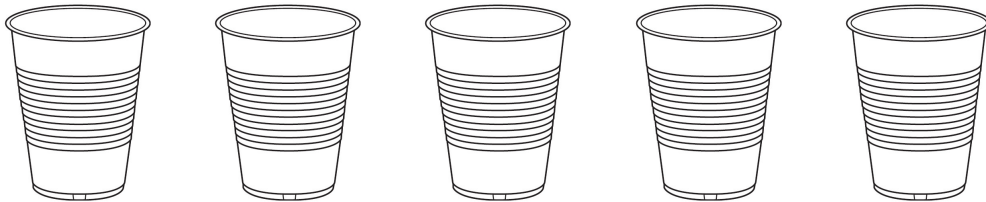
UW Math Circle
November 3, 2016

1. You have **three cups** in a row.



- (a) If the cups are labeled 1, 2 and 3, and they are all facing up, how many ways are there to arrange the cups in a line?
- (b) If the cups are labeled 1, 2 and 3, and they can be facing up or down, how many ways are there to arrange the cups in a line?
- (c) If the cups aren't labeled (and they all look identical), and they can be either facing up or facing down, how many arrangements of cups are possible?

2. Now, you have **five cups** in a row.



- (a) If the cups are labeled 1, 2, 3, 4 and 5, how many ways are there to arrange the cups in a line?
- (b) If the cups are not labeled (and look identical), how many ways are there to arrange the cups so that exactly two cups are facing down?
- (c) If the cups are not labeled (and look identical), how many ways are there to arrange the cups so that at least two cups are facing down?

3. You still have **five** cups in a row, all facing up.

(a) You are allowed to turn over **two cups** at a time. Is it possible that, after some number of turns, all of the cups are facing down?

(b) If you are allowed to turn over **three cups** at a time (instead of two), is it possible that, after some number of turns, all of the cups are facing down?

(c) In (a) or (b), if it *is possible* to get all of the cups facing down, what is the minimal number of moves it takes to get all of the cups facing down?

4. Now, you have n cups in a row, all facing up.

(a) If you can turn **two cups** over at a time, for what values of n is it possible to get all of the cups facing down?

(b) If you can turn **three cups** over at a time, for what values of n is it possible to get all of the cups facing down?

(c) If k the the number of cups you can turn over each time, for what values of n and k is it possible to get all of the cups facing down?