

UW Math Circle
October 20, 2016

1. The product of 22 integers is equal to 1. Can their sum be zero?
2. Can you find natural numbers a and b such that $ab(a - b) = 987654321$?
3. The numbers $1, 2, 3, \dots, 2017$ are written on a chalkboard. We erase any two numbers, and replace these two numbers with their positive difference (so if we erased 13 and 137, we would write 124 on the board). After doing this many times, we are left with one number on the board. Could this number be zero?
4. Three grasshoppers are playing leap frog on a line. At each turn, one grasshopper jumps over one other grasshopper (but not over the two other grasshoppers). Is it possible for the grasshoppers to return to their original configuration after 2016 turns? What about 2017 turns?
5. There are 100 campers at Camp Gauss, and after each meal 3 campers are in charge of washing the dishes. The campers take turn washing the dishes. Is it possible that at some point every camper has been on dish washing duty with every other camper exactly one time?



6. A 17 digit number is chosen and its digits are reversed, forming a new number. Show that the sum of these two numbers contains at least 1 even digit.