

Auction II

UW Math Circle

1. Make the largest number possible using 0, 1, 2, 3, 4 and the mathematical symbols as many times as you want:

+ - × ÷ √ · ()

Exponents are allowed, but stacked exponents (e.g. 4^{3^2}) are disallowed.

2. A word is called *descendable* if we can remove any number of letters from the right and get another word. For example, PINGS is descendable since PING, PIN, and PI are all words. Find the **longest possible descendable word**.



3. In standard chess theory, pawns are worth 1 point, knights and bishops 3, rooks 5, queens 9, and the king, of course ∞ . Using any combination of pieces of a single color, attack every square on the board using **as few points as possible**.

Note that a piece does not attack the square it stands on. For example, you could place 8 rooks on the same column for a total of 40 points.

4. In a computer game, your goal is to unlock numbers while saving money. Initially, only the number 1 is unlocked. You can combine unlocked numbers of the same size to unlock another number, at a cost of 1 dollar per number used. Once a number is unlocked, it is permanently unlocked.

For example, say I wanted to unlock the number 10. I could combine the number 1 ten times, at a cost of 10 dollars. Or, I could combine 1 five times, to unlock the number 5. Then, I combine 5 twice, and I've unlocked 10 for a cost of $5 + 2 = 7$ dollars.

Find **the cheapest way possible** to unlock both $1862 = 2 \times 19 \times 49$ and $2016 = 7 \times 9 \times 32$.



5. I have six boxes, each initially containing a single coin. You can perform two operations on these boxes:
- A: Remove a coin from a box and add two to the box after it.
 - B: Remove a coin from a box and switch the two boxes after it.
- Using these operations, get **as many coins as possible**.

