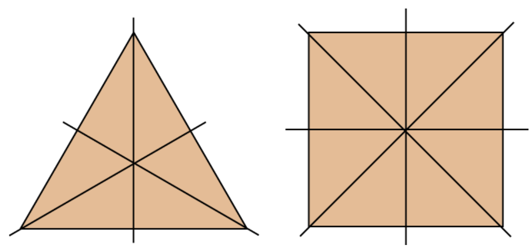
UW Math Circle
November 12 2015

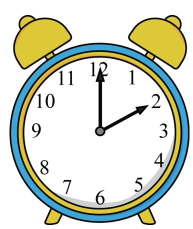
If you're reading this, that means today we learned what generators and orders are. Remember the *order of an element* x is the smallest integers n such that $x^n = e$. The *order of a group* is just the size of the group.

A group is *generated by a set* $S = (a_1, a_2, \ldots, a_n)$ if every element in the group is some combination of the elements of S. If S has a finite number of elements, G is said to be *finitely generated*. If S has just one element, G is called *cyclic*.

1. Show that D_n is generated by r and s, where r is a rotation, and s is a flip.



2. (From last time)
   (a) Let C_n denote the set of integers 0 through $n-1$, with the operation of addition mod n. Show that C_n is in fact a group.
   (b) Is it still a group under multiplication? If so, prove it. If not, can you fix it so that it becomes a group under multiplication?
   (c) Let $\otimes$ denote multiplication of integers mod 12. For example, $6 \otimes 7 = 6$ and $7 \otimes -9 = 9$. Find all sets S such that $(S, \otimes)$ is a group.



   (d) Show that C_n is cyclic for all n. It is called the (careful!) *cyclic group of order* n. We need to be careful because we don't know that it is the *only* cyclic group of order n. We might prove this later.

3. (a) If $(G, *)$ is a group, and S is a subset of G, and $(S, *)$ also happens to be a group, then $(S, *)$ is called a *subgroup* of G. Find all subgroups of C_3. How about C_4? C_6?
   (b) Find the order of all the elements of C_6. How about C_7? C_8?

4. (a) Show that $\mathbb{Q}$ is not cyclic.
   (b) Show that $\mathbb{Q}$ is not even finitely generated.