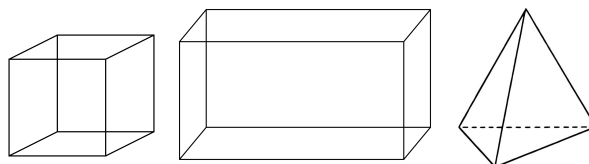# UW Math Circle
November 5 2015

For these problems, we will only consider rotational and reflective symmetry.
Try to do the problems in order, as they (should) get progressively more challenging.

1. Find all symmetries of a cube. What about a rectangular prism? A tetrahedron?

2. Consider the symmetries of a regular polygon. Call a rotation operation r, and a reflection operation s. What relations can you find between r and s?

3. Prove that symmetries over a regular polygon form a group. We call this the *dihedral group* of a polygon.
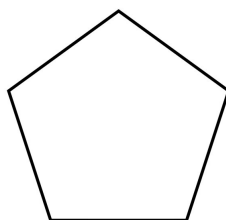
4. Is $(\mathbb{R}, +)$ a group? What about $(\mathbb{R}, \times)$? $(\mathbb{Z}, +)$? $(\mathbb{Z}, \times)$? If some of these are not groups, how could you make them groups?

5. Prove that the identity of a group is unique.

6. Remember that C_n denotes the set of integers 0 through $n - 1$, with the operation of addition mod n. Show that C_n is in fact a group.

7. Let $\otimes$ denote multiplication of integers mod 12. For example, $6 \otimes 7 = 6$ and $7 \otimes -9 = 9$. Find all sets S such that $(S, \otimes)$ is a group.