# UW Math Circle
October 1, 2015

1. I draw 1,000,000 points on a piece of paper, such that no three of them lie on the same line. Prove that I can draw a line that has exactly 500,000 points on either side.

2. A frog is moving from the lower left corner of a chessboard to the upper right corner. The frog can only jump up or to the right. How many paths can the frog take to its destination? What if the central 2×2 square is removed?

3. If n is a positive integer, is $2^n + 3^n$ ever a perfect square? (Hard) What if n could be negative? Is $2^n + 3^n$ ever the square of a rational number?

4. I have 7 of each type of tetromino. Can I rearrange them to make a 28×7 rectangle?
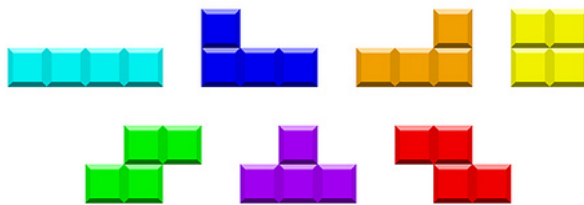
5. Alice and Bob are playing a game. They take turns choosing integers from 1 to 10. Once a number has been chosen by a player, it cannot be chosen again. A player wins when their chosen numbers sum to 15. If Alice goes first, who wins this game?
   The win condition is changed to the following: A player wins when any subset of their chosen numbers sums to 15. Now who wins?

6. Does there exist a (integral) power of 2, such that when you rearrange its digits, you get another power of 2?