

# UW Math Circle

March 10, 2016

**Instructions:** Kristin forgot what numbers are, so she decided to make up her own. Here are some numbers she invented:

123, 32154, 1423, 312, 2413, 162543

Austin decided to make up some numbers of his own, but Kristin told him that they don't count:

22, 243, 35, 541344, 1337

1. Which of these numbers count according to Kristin's definition of a number?

615324, 1327645, 31514, 12345, 541, 7, 41532, 3521, 312, 4132, 13542, 1, 14263675

2. (a) Write down all the one-digit numbers according to Kristin's rules. How many are there?

- (b) Write down all the two-digit numbers according to Kristin's rules. How many are there?

- (c) Write down all the three-digit numbers according to Kristin's rules. How many are there?

- (d) Write down all the four-digit numbers according to Kristin's rules. How many are there?

- (e) How many  $n$ -digit numbers are there according to Kristin's rules?

3. Next, Kristin made up rules for how to multiply two numbers together. Here are some of her results:

$$213 \cdot 321 = 231$$

$$2143 \cdot 4321 = 3412$$

$$21435 \cdot 54321 = 45231$$

$$214356 \cdot 654321 = 563421$$

$$2143567 \cdot 7654321 = 6745321$$

$$21435678 \cdot 87654321 = 78564321$$

$$321 \cdot 213 = 312$$

$$4321 \cdot 2413 = 3142$$

$$54321 \cdot 13524 = 42531$$

$$654321 \cdot 234651 = 156432$$

$$7654321 \cdot 2543671 = 1763452$$

$$1324 \cdot 3421 = 3241$$

$$1324 \cdot 3412 = 3142$$

$$1324 \cdot 3142 = 3412$$

$$1324 \cdot 1342 = 1432$$

$$1324 \cdot 1324 = 1234$$

$$1324 \cdot 1432 = 1342$$

$$123 \cdot 123 = 123$$

$$123 \cdot 321 = 321$$

$$123 \cdot 231 = 231$$

$$132 \cdot 231 = 213$$

$$213 \cdot 213 = 123$$

$$213 \cdot 321 = 231$$

$$213 \cdot 123 = 213$$

$$321 \cdot 321 = 123$$

$$231 \cdot 231 = 312$$

$$231 \cdot 213 = 132$$

$$23145 \cdot 41253 = 12453$$

$$3761542 \cdot 2617534 = 1432576$$

How would Kristin multiply these numbers:

$$312 \cdot 132 =$$

$$52143 \cdot 14235 =$$

$$21 \cdot 21 =$$

$$1347652 \cdot 3614527 =$$

$$4321 \cdot 2341 =$$

4. Can you find a number so that

$$31425 \cdot \underline{\hspace{2cm}} = 12345$$

$$\underline{\hspace{2cm}} \cdot 31425 = 12345$$