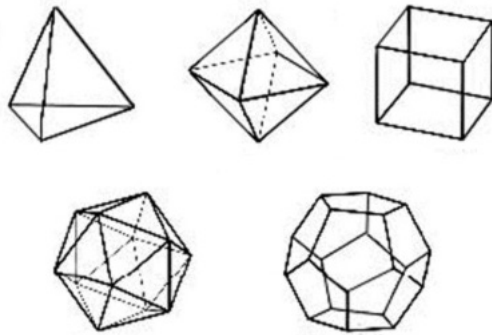


UW Math Circle  
February 25, 2016

Remember Euler's Formula: For a planar graph,  $V - E + F = 2$ , and the inequality  $E \leq 3V - 6$  (for planar graphs with more than 2 vertices).

1. There are 7 lakes in Lakeville, and the lakes are connected by 10 canals. You can travel by boat from any lake to every other lake via the canals. How many islands are there in Lakeland?
2. If every vertex in a planar graph has degree greater than or equal to 3, show that  $E \leq 3F - 6$ . Note that this is  $F$  instead of  $V$ !
3. A heptagon (a 7-sided polygon) is divided into convex pentagons and hexagons so that each vertex of the 7-gon is part of at least 2 smaller polygons. Show that there are at least 13 pentagons in this division of the 7-gon.
4. A polyhedron is a three dimensional shape made out of polygons glued along their edges. There are special types of polyhedrons called Platonic Solids. These are convex (no cavities or holes) polyhedra where each face is the same regular polygon, and at each vertex the same number of faces come together. See the picture below.



- (a) Show that the vertices and edges of any convex polyhedron give a planar graph.
- (b) If a platonic solid is made of a regular polygon with  $p$  sides and where  $q$  faces come together at each vertex, what are the possible values for  $p$  and  $q$ ?
- (c) Conclude that the platonic solids listed above are all the possible platonic solids— there are only 5!

5. Each edge of the complete graph on 11 vertices is colored either red or blue (the complete graph on 11 vertices is the graph with 11 vertices and 1 edge connecting every vertex to every other one). Show that either the graph consisting of all the red edges or the graph consisting of all the blue edges is nonplanar.