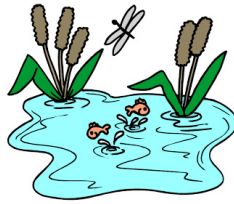


UW Math Circle
February 25, 2016
Homework

On some of these problems, you may want to remember Euler's Formula: for a planar graph, $V - E + F = 2$, and the inequality $E \leq 3V - 6$ (for planar graphs with more than 2 vertices).

1. In Pondtown there are ponds connected by canals, and each pond is connected to at least 5 other ponds. Is it possible that there are 10 ponds in Pondtown?



2. Austin just returned from Fibtopia, where he claims that there are many lakes connected by rivers. He says that three rivers flow in to every lake and that four rivers flow out of every lake. Can he be right?
3. There are 20 points inside a square. The points are connected by nonintersecting lines with each other and with the vertices of the square so that the square is divided into triangles.

How many triangles are there?

4. (a) Each of the edges of the complete graph on 9 vertices is colored either blue or red. Prove that there exists four vertices with all the edges connected to them blue, or three vertices with all the edges connected to them red.
(b) Each of the edges of a complete graph with 10 vertices is colored either blue or red. Prove that there are four vertices such that all the edges connected to them are the same color.

