# UW Math Circle

January 28, 2016

Homework

**The Pigeonhole Principle:** If $N+1$ pigeons fly into $N$ birdhouses, then some birdhouse will contain at least 2 pigeons.

**The Generalized Pigeonhole Principle:** If $k \cdot N + 1$ pigeons fly into $N$ birdhouses, then some birdhouse will contain at least $k + 1$ pigeons.

1. How many cards would you need to pick from a standard deck to guarantee you have at least one pair?

2. A $10 \times 10$ table is filled in with positive integers so that adjacent integers *(integers are adjacent if their squares share a side)* differ by 5 or less. Show that the table must contain two identical integers.

3. Prove there is an integer consisting only of 1's (something like 11111...) that is divisible by 7373.

4. There are $n^2+1$ boxes in a line. On each box I write a different number. Prove that you can walk down the line of boxes and pick n+1 boxes where either the number on each box you pick is smaller than the number on the previous box you picked, or where the number on each box you pick is greater than the number on the previous box you picked.

5. A mural on a very large wall is painted with three colors: grey, green, and blue. Prove that no matter how the wall is painted, there are two points of the same color exactly one foot apart.