UW Math Circle
January 21, 2016
Homework

The Pigeonhole Principle: If $N+1$ pigeons fly into N birdhouses, then some birdhouse will contain at least 2 pigeons.

The Generalized Pigeonhole Principle: If $k \cdot N + 1$ pigeons fly into N birdhouses, then some birdhouse will contain at least $k + 1$ pigeons.

1. Prove that among any three integers, there are two whose sum is even.

2. Show that if there is a party with two or more people, then at least two people have the same number of friends at the party. (We say that friendship is symmetric, so if Bob is Dave's friend, then Dave is also Bob's friend).

3. Jessica has a new backpack with 10 pockets in it, and she wants to put 44 things in it. Can she do this in such a way that each pocket has a different number of things?

4. Every square of a 2014 x 2014 chessboard is colored one of 2013 colors. In a given row or column, if two squares have the same color, then the entire row/column can be recolored that color. Is it possible to make the entire board one solid color?

5. Show that if you place 41 rooks on a 10×10 chessboard, then there will be at least five of them that do not attack one another.