

UW Math Circle
May 19th, 2016



1. A cruel jailer has four prisoners. The jailer places one of the prisoners in a room by herself, and the three others in a line, where they cannot see the people behind them. The jailer then places hats on all the prisoners' heads— two white hats and two black hats. A prisoner cannot see the hat on their own head. The jailer says that all four prisoners can go free if any one of the prisoners can name the color of the hat on their own head, but that if anyone says the wrong color all four will be immediately executed! Can the prisoners find a way to win their freedom?
2. Five escaped inmates have stolen 1,000 gold coins, and must find a way to distribute them. They are standing in a line. The criminals decide that they will divide the coins as follows: the first in line will propose a distribution of the coins, and then all the convicts will vote on it. If her plan gets a majority of the votes then it is followed, but if it isn't she is to be immediately killed, and then the procedure will start again with the four remaining prisoners (a tie vote doesn't count as a majority). The escaped inmates all want to get as many coins as possible.

Is there any way for the first convict to survive? If so, what is the maximum number of coins she can get?
3. Three of the inmates are recaptured, and the sympathetic warden says he will devise a *fair* test to choose someone to go free. He says that the first person to guess the color of the hat on their head will go free. The warden then shows the three inmates 5 hats - two white and three black. Then he turns off the lights in the room and puts a hat on each prisoner's head. After that the warden hides the remaining two hats, but before he could turn the lights back on, one of the prisoners announced the color of their hat. And they were right! How did they do it?
4. Thomas and Austin are in prison, having been accused of dividing by zero. They are placed in solitary confinement, and each has the opportunity to betray the other or to stay silent. If each betrays the other, they both will get two years in jail. If both stay silent, they will each get one year in jail. If Austin betrays Thomas but Thomas stays silent, Thomas will get 3 years in jail and Austin will go free (and vice versa). Austin and Thomas cannot communicate after they are in prison. What should they do?

5. Ten prisoners are randomly given hats, either red or blue, and placed in a line where they can see the hats of the people in front of them, but not their own. Starting at the back of the line, the prisoners must say either "Red" or "Blue". If they say the color of their hat they are released immediately, but if they say the wrong color they are immediately executed. What is a strategy that guarantees the best chance of survival for the most prisoners?
6. Now the prisoners can see everyone's hat but their own, but they must all say either "Red" or "Blue" at the same time. If they say the color of their hat, they are let go. What is a strategy that guarantees at least 5 of them will be let go? Does there exist a strategy that guarantees more than five will be let go?
7. There are 10 prisoners (each numbered 1 to 10) and 10 boxes in a line. There is one number in each box. The prisoners do not know what number is in what box, and they will go into the room with boxes one at a time to open up to 5 boxes to attempt to find their number. If each prisoner finds their own number, they will all go free. Find a strategy that has the best probability of success.