UW Math Circle

April 7, 2016

Something went wrong in the following proofs, and it's up to you to find out what happened!

1. *All Powers of 2 Are Equal to 1*

   We are going to prove by induction that,

   $$\text{For all integers } n \geq 0, \quad 2^n = 1$$

   We start with the base case $n = 0$; we know $2^0 = 1$.

   Now for the induction step, assume the equation is correct for all $n \leq k$, that is

   $$2^0 = 1, 2^1 = 1, 2^2 = 1, ..., 2^k = 1.$$

   From these we now derive that also $2^{k+1} = 1$:

   $$2^{k+1} = \frac{2^{2k}}{2^{k-1}} = \frac{2^k \times 2^k}{2^{k-1}} = \frac{1 \times 1}{1} = 1$$

   Induction is complete. *What went wrong?*

2. *We prove that all cats are of the same color!*

   We start with the $n = 1$ case. If there is only one cat, then it is the same color as all other cats, because there is only one cat!

   For the induction step, suppose the claim is true for any n: any n cats are the same color. Then, consider any group of $n + 1$ cats, labeled: $\{c_1, c_2, ..., c_n, c_{n+1}\}$. If we remove the last cat c_{n+1}, then the cats $\{c_1, c_2, ..., c_n\}$ are the same color by the Induction Hypothesis (for example, white). And, if we also took cat c_1 out, then the remaining cats $\{c_2, ..., c_n\}$ are still all white, and c_1 is white as well.

   Now let us put the last cat c_{n+1} into the above group: the group $\{c_2, ..., c_n, c_{n+1}\}$ now has n cats, so they are the same color by the Induction Hypothesis (still white).

   So, the cats $\{c_1, c_2, ..., c_n, c_{n+1}\}$ are the same color.

   Induction is complete. *What went wrong?*