UW Math Circle

June 2nd, 2016

Practice Olympiad

1. Is it possible to draw some number of diagonals in a convex hexagon so that every diagonal crosses EXACTLY three others in the interior of the hexagon? (Diagonals that touch at one of the corners of the hexagon DO NOT count as crossing.)

2. A 3×3 square grid is filled with positive numbers so that

   (a) the product of the numbers in every row is 1,

   (b) the product of the numbers in every column is 1,

   (c) the product of the numbers in any of the four 2×2 squares is 2.

   What is the middle number in the grid? Find all possible answers and show that there are no others.

3. You walk into a room and find five boxes sitting on a table. Each box contains some number of coins, and you can see how many coins are in each box. In the corner of the room, there is a large pile of coins. You can take two coins at a time from the pile and place them in different boxes. If you can add coins to boxes in this way as many times as you like, can you guarantee that each box on the table will eventually contain the same number of coins?

4. Students from Hufflepuff and Ravenclaw were split into pairs consisting of one student from each house. The pairs of students were sent to Honeydukes to get candy for Fathers Day. For each pair of students, either the Hufflepuff student brought back twice as many pieces of candy as the Ravenclaw student or the Ravenclaw student brought back twice as many pieces of candy as the Hufflepuff student. When they returned, Professor Trelawney determined that the students had brought back a total of 1000 pieces of candy. Could she have possibly been right? Why or why not? Assume that candy only comes in whole pieces (cannot be divided into parts).

5. Alex, Bob and Chad are playing a table tennis tournament. During each game, two boys are playing each other and one is resting. In the next game the boy who lost a game goes to rest, and the boy who was resting plays the winner. By the end of tournament, Alex played a total of 10 games, Bob played 15 games, and Chad played 17 games. Who lost the second game?

6. You have a deck of 50 cards, each of which is labeled with a number between 1 and 25. In the deck, there are exactly two cards with each label. The cards are shuffled and dealt to 25 students who are sitting at a round table, and each student receives two cards. The students will now play a game. On every move of the game, each student takes the card with the smaller number out of his or her hand and passes it to the person on his/her right. Each student makes this move at the same time so that everyone always has exactly two cards. The game continues until some student has a pair of cards with the same number. Show that this game will eventually end.

7. Consider a set of finitely many points on the plane such that if we choose any three points A, B, C from the set, then the area of the triangle ABC is less than 1. Show that all of these points can be covered by a triangle whose area is less than 4.

8. A palindrome is a number that is the same when read forward and backward. For example, 1771 and 23903030932 are palindromes. Can the number obtained by writing the numbers from 1 to n in order be a palindrome for some $n > 1$? (For example, if n = 11, the number obtained is 1234567891011, which is not a palindrome.)

9. A computer starts with a given positive integer to which it randomly adds either 54 or 77 every second and prints the resulting sum after each addition. For example, if the computer is given the number 1, then a possible output could be: 1, 55, 109, 186, Show that after finitely many seconds the computer will print a number whose last two digits are the same.