UW Math Circle
December 3, 2015
Homework

1. Show that 3333333333333333333333333335 is not the square of an integer.

2. What is the last digit of 137^{731}? What is the last digit of 3412^{2143}?

3. Use modular arithmetic to show that if 3 does not divide n, then 3 does not divide n^2. Conclude that if $3|n^2$, then $3|n$.

4. Call a positive integer n triangluar if $n = \frac{1}{2}(k)(k+1)$, where k is some other positive integer. The first few triangular numbers are $1, 3, 6, 10, \ldots$.

    (a) Show that if $k \equiv r \pmod 9$, then $\frac{1}{2}(k)(k+1) \equiv \frac{1}{2}(r)(r+1) \pmod 9$.

    (b) Determine a criteria to determine that a number is not triangular.

    (c) Find infinitely many numbers that are are not the sum of two triangular numbers.

1   3   6   10   15