UW Math Circle

November 19, 2015

Homework

1. Show that a number is divisible by 9 if and only if the sum of its digits is divisible by 9.

2. Show that a number is divisible by 11 if and only if the alternating sum of its digits is divisible by 11. ("Alternating sum" means adding the first digit, subtracting the second, adding the third, subtracting the fourth, and so on; i.e. 209 is divisible by 11 ($19 \times 11 = 209$) and the alternating sum of its digits is $2 - 0 + 9 = 11$.)

3. Show that if a, b, and c are integers for which $a + b + c$ is divisible by 6, then $a^3 + b^3 + c^3$ is divisible by 6, too.

4. A set of numbers (a, b, c) with $a^2 + b^2 = c^2$ is called a *Pythagorean Triple*.

   (a) Show that in a Pythagorean triple, at least one of the numbers is divisible by 3.

   (b) Show that in a Pythagorean triple, at least one of the numbers is divisible by 5.