

UW Math Circle
November 12, 2015

1. You go to a store and ask to change a 5 dollar bill into coins. The clerk gives you pennies, nickels, and quarters. You count that the clerk gives you 31 coins. Is it possible that the clerk gave you 5 dollars?



2. Caleb has a book with 96 pages, numbered 1 through 192. He then tears out 25 pages, and adds up the 50 page numbers. Could he get 2016 as the sum of the 50 numbers?

3. Do there exist natural numbers a and b with $ab(a - b) = 987654321$?

4. The numbers 1 – 2018 are written on a board, and play a game where you erase any 2 numbers and write their difference on the board. You do this repeatedly until there is only 1 number left on the board. Is this number even or odd?

5. Mark is designing a video game that takes place on a 200 by 200 grid. Two players start on opposite corners of the grid, and on each move, they can either jump 3 squares left/right and 4 squares up/down or 2 squares left/right and 2 squares up/down. Mark knows his game will crash if the players ever land on the same square, so he asks you for help: is it possible for the two players to end up on the same square after some number of moves?



6. Jill and Sue each make a square out of 1 inch by 1 inch tiles. Is it possible that the total area of their two squares is 2015in^2 ? What about 20000000015 ?